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METHODOLOGIES AND APPLICATION

# A PROMETHEE-based outranking method for multiple criteria decision analysis with interval type-2 fuzzy sets

**Ting-Yu Chen**



**Abstract** This paper develops new methods based on the preference ranking organization method for enrichment evaluations (PROMETHEE) that use a signed distance-based approach within the environment of interval type-2 fuzzy sets for multiple criteria decision analysis. The theory of interval type-2 fuzzy sets provides an intuitive and computationally feasible way of addressing uncertain and ambiguous information in decision-making fields. Many studies have developed multiple criteria decision analysis methods in the context of interval type-2 fuzzy sets; most of these methods can be characterized as scoring or compromising models. Nevertheless, the extended versions of outranking methods have not been thoroughly investigated. This paper establishes interval type-2 fuzzy PROMETHEE methods for ranking alternative actions among multiple criteria based on the concepts of signed distance-based generalized criteria and comprehensive preference indices. We develop interval type-2 fuzzy PROMETHEE I and interval type-2 fuzzy PROMETHEE II procedures for partial and complete ranking, respectively, of the alternatives. Finally, the feasibility and applicability of the proposed methods are illustrated by a practical problem of landfill site selection. A comparative analysis is also performed with ordinary fuzzy PROMETHEE methods to validate the effectiveness of the proposed methodology.

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T.-Y. Chen (✉)
Department of Industrial and Business Management, Graduate Institute of Business and Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan
e-mail: tychen@mail.cgu.edu.tw

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## 1 Introduction

The preference ranking organization method for enrichment evaluations (PROMETHEE), which was introduced by Brans (1982), is a well-known and widely used outranking method for multiple criteria decision analysis (MCDA). Considering the selection of a finite set of alternative actions among (conflicting) criteria, the PROMETHEE methods incorporate pairwise comparisons and outranking relationships for selection of the best criteria. PROMETHEE is also a simple ranking method for conception and application compared with other methods for solving multiple-criteria evaluation problems (Brans et al. 1986). The PROMETHEE methods compute positive and negative preference flows for each alternative and facilitate the selection of a final alternative by the decision maker (Peng et al. 2011). The positive preference flow indicates how an alternative outranks all other alternatives, and the negative preference flow indicates how an alternative is outranked by all other alternatives (Brans and Mareschal 2005). PROMETHEE has been successfully applied to many MCDA problems; its effectiveness is due to its solid mathematical properties and its ease of use (Brans and Mareschal 2005; Behzadian et al. 2010; Hsu and Lin 2012).

The PROMETHEE family of outranking methods encompasses PROMETHEE I for partial ranking of alternatives (Brans 1982), PROMETHEE II for complete ranking of alternatives (Brans 1982), PROMETHEE III for ranking of alternatives based on intervals (Brans et al. 1984), PROMETHEE IV for continuous cases (Brans et al. 1984), PROMETHEE V

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for problems with segmentation constraints (Brans and Mareschal 1992), PROMETHEE VI for human brain representation (Brans and Mareschal 1995), PROMETHEE GAIA for geometrical analysis of interactive aid (Mareschal and Brans 1988; Brans and Mareschal 1994), PROMETHEE GDSS for a group decision support system (Macharis et al. 1998), PROMETHEE TRI for dealing with sorting problems (Figueira et al. 2004), PROMETHEE CLUSTER for nominal classification (Figueira et al. 2004), and PROMETHEE GKS for robust ordinal regression (Kadziński et al. 2012), among other methods. Currently, methodologies of the PROMETHEE family and their applications have attracted considerable attention in the multiple criteria decision-making field.

The PROMETHEE methodologies have been extended to the fuzzy environment. Fernandez-Castro and Jimenez (2005) presented a methodological contribution related to PROMETHEE V, which suggests that some constraints are soft and some coefficients are estimated by fuzzy numbers. Zhang et al. (2009) used fuzzy PROMETHEE to conduct a comparative approach for ranking contaminated sites based on a risk assessment paradigm. Considering the fuzziness in the decision data, Li and Li (2010) presented a new extension of PROMETHEE II based on generalized fuzzy numbers. Chen et al. (2011) presented a fuzzy PROMETHEE method to evaluate outsourcing suppliers. By employing fuzzy numbers, Yilmaz and Dağdeviren (2011) combined F-PROMETHEE and a zero-one goal programming model to address the problem of equipment selection. Abedi et al. (2012) considered fuzzy scores expressed by fuzzy membership functions to explore porphyry copper deposits using PROMETHEE II. Hsu and Lin (2012) combined the concepts of fuzzy sets to represent the uncertain information in intrinsic risks with PROMETHEE to explore group package tours based on risk perception. Taha and Rostam (2012) developed a hybrid fuzzy AHP-PROMETHEE decision support system for machine tool selection.

Uncertain and imprecise assessment information is common in many practical MCDA situations because certain decision makers may express their judgments using linguistic terms (Hatami-Marbini and Tavana 2011). Due to a lack of data, time pressure, or decision makers' limited attention and information-processing capabilities, the decision makers often make their decisions within linguistic environments in real-world problems (Su 2011; Chen 2012a; Rajpathak et al. 2012). In this regard, interval type-2 fuzzy sets (IT2FSs) are very useful for conveniently modeling impressions and quantifying the ambiguous nature of linguistic judgments. IT2FSs efficiently express linguistic evaluations because they provide great flexibility to present uncertainties (Chen and Lee 2010a; Zhang and Zhang 2013). Based on the interval type-2 fuzzy framework, Chen and Chen (2009); Wei and Chen (2009), and (Chen 2011a,b, 2012a,b) presented a type-2 fuzzy linguistic system that contains nine-point linguistic rating scales and the corresponding interval type-2 trapezoidal fuzzy numbers (IT2TrFNs) required for measuring the importance weights and the alternative ratings. Other useful type-2 fuzzy linguistic systems for IT2FSs include seven-point scales (Chen and Lee 2010a; Hosseini and Tarokh 2011; Chen et al. 2012; Gilan et al. 2012; Wang et al. 2012), five-point scales (Chen and Lee 2010b; Hosseini and Tarokh 2011), four-point scales (Chen and Lee 2010b), and three-point scales (Chen and Lee 2010b; Hosseini and Tarokh 2011; Zhai and Mendel 2011; Han and Mendel 2012). With the aid of type-2 fuzzy linguistic systems, the IT2FS theory has been conveniently applied in practical multiple criteria decision-making problems (Zhai and Mendel 2011; Chen 2012b; Gilan et al. 2012; Wang et al. 2012).

Most extensions of the fuzzy PROMETHEE methods, such as the employment of fuzzy numbers, have been discussed within the decision environment of ordinary fuzzy sets. However, little attention has been given to the development of PROMETHEE methods based on IT2FSs. Many useful methods have been proposed for solving various MCDA problems (Hosseini and Tarokh 2011; Chen 2011a,b, 2012a,b; Acampora et al. 2012; Chen et al. 2012; Wang et al. 2012). For example, Chen (2011b) presented an integrated approach to combining objective and subjective importance values of criteria with IT2FSs. Wang et al. (2012) developed an approach to handling multiple criteria group decision-making problems in which the criterion values are characterized by IT2FSs and the information about the criterion weights is partially known. Chen (2012b) constructed an integrated programming model to estimate the optimal criterion weights from incomplete and inconsistent preference information and to determine the closeness coefficient values for therapy rankings in cancer care. Das et al. (2012) improved the accuracy of a fuzzy expert decision-making system by tuning the parameters of type-2 sigmoid membership functions of fuzzy input variables and determining the most appropriate membership function; they then applied the proposed method to a medical diagnostic decision-making system. Chen et al. (2013) developed an extended QUALIFLEX method for handling MCDA problems in the context of IT2FSs and applied it to a medical decision-making problem. Baležentis and Zeng (2013) extended MULTIMOORA by using generalized interval-valued trapezoidal fuzzy numbers to facilitate group decision making in the IT2FS framework. Zhang and Zhang (2013) used trapezoidal interval type-2 fuzzy soft sets to propose a multiple criteria group decision-making method. A number of studies have developed MCDA methods in the context of IT2FSs, but the PROMETHEE methodologies are less developed in IT2FS settings. Considering the usefulness of type-2 fuzzy linguistic systems in decision making, we conducted this study to construct new interval type-2 fuzzy PROMETHEE methods, which are different from the existing MCDA methods within the IT2FS environment (e.g., the integrated programming models introduced by Chen 2011b, 2012b). On the basis of the IT2FS framework, this paper employs the popular fuzzy numbers with trapezoidal forms (as employed by Baležentis and Zeng 2013; Chen et al. 2013; Zhang and Zhang 2013, etc.), here named IT2TrFNs, to establish core PROMETHEE procedures using the concept of signed distances between IT2TrFNs.

The purpose of this paper is to develop interval type-2 fuzzy PROMETHEE methods for managing MCDA problems within the IT2FS environment. Classical PROMETHEE is generally implemented according to three main procedures: construction of generalized criteria, determination of an outranking relation on the set of alternatives, and evaluation of this relation to determine the priority order of the alternatives (Brans et al. 1984). Because the aggregation operations of uncertain linguistic terms can be determined through the operations of trapezoidal fuzzy numbers (Suo et al. 2012), we employ a type-2 fuzzy rating system (Chen 2011a,b, 2012a) consisting of nine-point linguistic rating scales and their corresponding IT2TrFNs to measure importance weights and alternative ratings. In addition to considering the context of IT2FSs in this study, we apply the concept of signed distances to establish new preference functions. Furthermore, several basic signed distance-based generalized criteria, which consist of the usual criterion, U-shaped criterion, V-shaped criterion, level criterion, V-shaped with indifference criterion, and Gaussian criterion, are provided to facilitate the determination of signed distance-based comprehensive preference indices (i.e., multiple criteria preference indices). Using an interval type-2 fuzzy framework, this study employs IT2TrFNs to propose extended definitions of leaving flows, entering flows, and net flows for the construction of relevant measures of outranking and outranked relations. Because PROMETHEE I and II are the most widely used methods among the PROMETHEE methodologies, this study establishes the interval type-2 fuzzy PROMETHEE I and interval type-2 fuzzy PROMETHEE II methods for partial ranking and complete ranking, respectively, of the alternatives. Finally, the feasibility and applicability of the proposed interval type-2 fuzzy PROMETHEE I and II methods are examined with a practical MCDA problem of landfill site selection. We also conduct a comparative analysis with ordinary fuzzy PROMETHEE methods to validate the effectiveness of the developed method.

This paper is organized as follows: Section 2 formulates an MCDA problem within an IT2FS framework and describes the concept of signed distances. Section 3 develops interval type-2 fuzzy PROMETHEE I and II outranking methods to handle MCDA problems with IT2TrFNs. Section 4 demonstrates the feasibility and applicability of the proposed methodology by applying it to a landfill site selection problem and conducting a comparative analysis with fuzzy PROMETHEE. Section 5 presents the conclusions. The concepts of IT2FSs and IT2TrFNs are used extensively throughout this paper. Several relevant definitions and operations of IT2FSs are briefly reviewed in the Appendix.

## 2 Preliminaries

This section establishes an MCDA problem within the IT2FS environment. It also describes the concept of signed distances among IT2TrFNs and provides a signed distance-based approach for determining the ordering of IT2TrFN values.

### 2.1 An MCDA problem defined for IT2FSs

The methods for evaluating alternatives and providing preference information about criteria are often guided by the subjective judgments of the decision maker. Most linguistic scales used in the context of IT2FSs (Chen and Lee 2010b; Hosseini and Tarokh 2011; Chen 2011a,b, 2012a; Chen et al. 2012; Gilan et al. 2012; Han and Mendel 2012; Wang et al. 2012) are based on a unipolar univariate model. Therefore, nonnegative IT2FSs are employed throughout this paper as a result of the common use of a unipolar setting in the linguistic rating system. In this paper, we adopt the nine-point linguistic rating scales presented by Chen (2011a, 2012a) to achieve better sensitivity when measuring variability in responses, as shown in Table 1. According to Chen (2011a, 2012a), there are nine translations of linguistic terms into IT2TrFNs; thus, the linguistic variables can be easily converted to IT2TrFNs.

Consider the following MCDA problem in which the ratings of alternative evaluations and criterion importance are expressed as IT2TrFNs. First, define the alternative set

**Table 1** Linguistic variables and their corresponding IT2TrFNs

| Linguistic terms | IT2TrFNs |
|---|---|
| Absolutely high (AH) | [(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)] |
| Very high (VH) | [(0.9475, 0.985, 0.9925, 0.9925; 0.8), (0.93, 0.98, 1.0, 1.0; 1.0)] |
| High (H) | [(0.7825, 0.815, 0.885, 0.9075; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)] |
| Medium high (MH) | [(0.65, 0.6725, 0.7575, 0.79; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)] |
| Medium (M) | [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)] |
| Medium low (ML) | [(0.2325, 0.255, 0.325, 0.3575; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)] |
| Low (L) | [(0.0875, 0.12, 0.16, 0.1825; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)] |
| Very low (VL) | [(0.0075, 0.0075, 0.015, 0.0525; 0.8), (0.0, 0.0, 0.02, 0.07; 1.0)] |
| Absolutely low (AL) | [(0.0, 0.0, 0.0, 0.0; 1.0), (0.0, 0.0, 0.0, 0.0; 1.0)] |

$Z = \{z_1, z_2, \ldots, z_m\}$ from which the decision maker must choose. Next, define $X = \{x_1, x_2, \ldots, x_n\}$ as the criterion set that contains the criteria by which the alternative performances are measured. The set $X$ can generally be divided into two sets, $X_b$ and $X_c$, where $X_b$ denotes a collection of benefit criteria (i.e., larger values of $x_j$ indicate a greater preference); $X_c$ denotes a collection of cost criteria (i.e., smalle values of $x_j$ indicate a greater preference); and $X_b \cap X_c = \varnothing$ and $X_b \cup X_c = X$.

Each alternative is evaluated with respect to each of the $n$ criteria that is based on the experience and subjective judgments of the decision maker, and the assessment is expressed as a non-negative IT2TrFN. Let $A_{ij}$ denote the evaluative rating of alternative $z_i \in Z$ with respect to criterion $x_j \in X$ with $A_{ij}$ expressed as

$$A_{ij} = [A_{ij}^L, A_{ij}^U] = \left[\left(a_{1ij}^L, a_{2ij}^L, a_{3ij}^L, a_{4ij}^L; h_{ij}^L\right), \left(a_{1ij}^U, a_{2ij}^U, a_{3ij}^U, a_{4ij}^U; h_{ij}^U\right)\right], \tag{1}$$

where $0 \le a_{1ij}^L \le a_{2ij}^L \le a_{3ij}^L \le a_{4ij}^L$, $0 \le a_{1ij}^U \le a_{2ij}^U \le a_{3ij}^U \le a_{4ij}^U$, $a_{1ij}^U \le a_{1ij}^L$, $a_{4ij}^L \le a_{4ij}^U$, and $0 < h_{ij}^L \le h_{ij}^U \le 1$ (see Fig. 1). In this context, $A_{ij}^L = (a_{1ij}^L, a_{2ij}^L, a_{3ij}^L, a_{4ij}^L; h_{ij}^L)$ and $A_{ij}^U = (a_{1ij}^U, a_{2ij}^U, a_{3ij}^U, a_{4ij}^U; h_{ij}^U)$ denote the lower and upper extremes, respectively, of the IT2TrFN $A_{ij}$, where $A_{ij}^L \subset A_{ij}^U$. For $i = 1, 2, \ldots, m$, the characteristic of alternative $z_i$ is represented in the following manner:

$$A_i = \left\{\left\langle x_j, A_{ij}\right\rangle \middle| x_j \in X, j = 1, 2, \ldots, n\right\}. \tag{2}$$

Similarly, the importance weight $W_j$ of criterion $x_j \in X$ provided by the decision maker is expressed as

$$W_j = [W_j^L, W_j^U] = \left[\left(w_{1j}^L, w_{2j}^L, w_{3j}^L, w_{4j}^L; h_j^L\right), \left(w_{1j}^U, w_{2j}^U, w_{3j}^U, w_{4j}^U; h_j^U\right)\right], \tag{3}$$

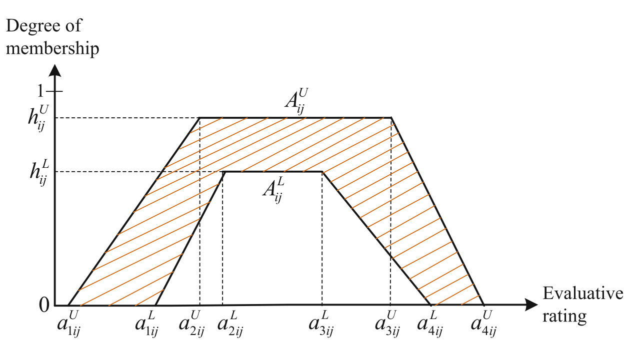


**Fig. 1** A geometrical interpretation of an IT2TrFN $A_{ij}$

where $0 \le w_{1j}^L \le w_{2j}^L \le w_{3j}^L \le w_{4j}^L$, $0 \le w_{1j}^U \le w_{2j}^U \le w_{3j}^U \le w_{4j}^U$, $w_{1j}^U \le w_{1j}^L$, $w_{4j}^L \le w_{4j}^U$, and $0 < h_j^L \le h_j^U \le 1$. Additionally, $W_j^L = (w_{1j}^L, w_{2j}^L, w_{3j}^L, w_{4j}^L; h_j^L)$ and $W_j^U = (w_{1j}^U, w_{2j}^U, w_{3j}^U, w_{4j}^U; h_j^U)$, where $W_j^L \subset W_j^U$. An IT2TrFN $W$ is defined as follows:

$$W = \left\{\left\langle x_j, W_j\right\rangle \middle| x_j \in X, j = 1, 2, \ldots, n\right\}. \tag{4}$$

## 2.2 Signed distance-based approach

In this study, we use a simple and effective procedure that is based on signed distances to define the ordering of IT2TrFNs. The concept of signed distances, which are also referred to as oriented distances or directed distances, can be used to determine rankings of fuzzy numbers (Chiang 2001; Chen and Ouyang 2006). Despite the multiple ranking methods, no decision maker is able to consistently rank fuzzy numbers by using human intuition in all cases (Abbasbandy and Asady 2006). Certain limitations were discovered when ranking fuzzy numbers by the following methods: the coefficient of variation, the distance between fuzzy sets, the centroid point and the original point, and the weighted mean value (Yao and Wu 2000; Abbasbandy and Asady 2006). The signed distance method is capable of effectively ranking various fuzzy numbers and their images (Yao and Wu 2000). The signed distance method calculations are also less complicated than the signed distance method calculations of other approaches (Abbasbandy and Asady 2006). The signed distance method can use both positive and negative values to define the ordering of fuzzy numbers. Therefore, this paper employs a signed distance-based approach to compare the IT2TrFN values.

Consider the IT2TrFN rating $A_{ij}$ of the alternative $z_i$ on the criterion $x_j$. Let $A_{ij}(\alpha)$ $(=[A_{ij}^L(\alpha), A_{ij}^U(\alpha)])$ be the intervals of confidence for the level of presumption of $\alpha$ (i.e., the $\alpha$-cut), where $\alpha \in [0, 1]$. The $\alpha$-cut of $A_{ij}$ is denoted as

$$A_{ij}(\alpha) = \begin{cases} \left[[{}^lA_{ij}^L(\alpha), {}^rA_{ij}^L(\alpha)], [{}^lA_{ij}^U(\alpha), {}^rA_{ij}^U(\alpha)]\right] & \text{if } 0 \le \alpha < h_{ij}^L, \\ \left[{}^lA_{ij}^U(\alpha), {}^rA_{ij}^U(\alpha)\right] & \text{if } h_{ij}^L \le \alpha \le h_{ij}^U, \end{cases} \tag{5}$$

where ${}^lA_{ij}^L(\alpha)$ and ${}^lA_{ij}^U(\alpha)$ are the left-hand points of the $\alpha$-cut, and ${}^rA_{ij}^L(\alpha)$ and ${}^rA_{ij}^U(\alpha)$ are the right-hand points of the $\alpha$-cut. Figures 2 and 3 provide convenient geometric interpretations of the left- and right-hand points of the $\alpha$-cut of $A_{ij}$ for $0 \le \alpha < h_{ij}^L$ and $h_{ij}^L \le \alpha \le h_{ij}^U$, respectively. Let $\xi \in \{L, U\}$. For each $\xi$, ${}^lA_{ij}^{\xi}(\alpha)$ and ${}^rA_{ij}^{\xi}(\alpha)$ are calculated by the following:

$${}^lA_{ij}^{\xi}(\alpha) = a_{1ij}^{\xi} + \frac{(a_{2ij}^{\xi} - a_{1ij}^{\xi})\alpha}{h_{ij}^{\xi}}, \tag{6}$$

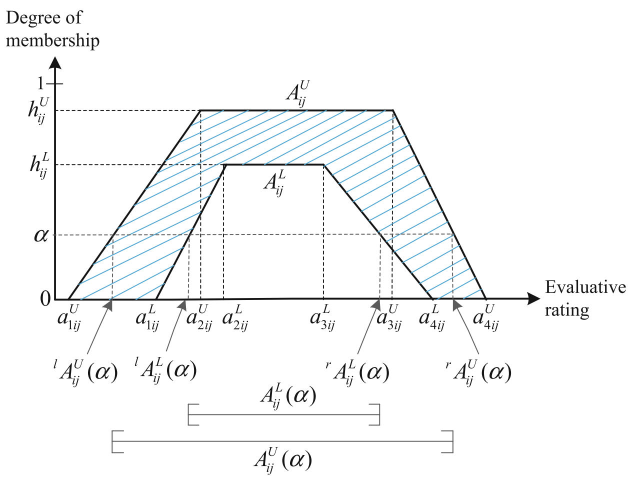


**Fig. 2** The $\alpha$-cut of an IT2TrFN $A_{ij}$ for $0 \le \alpha < h_{ij}^L$.

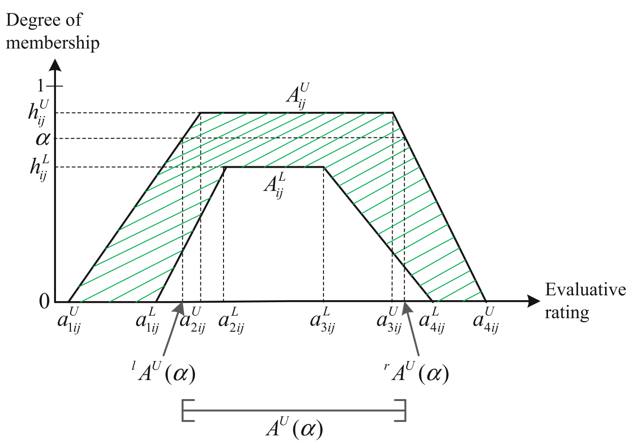


**Fig. 3** The $\alpha$-cut of an IT2TrFN $A_{ij}$ for $h_{ij}^L \le \alpha \le h_{ij}^U$.

$$
{}^rA_{ij}^{\xi}(\alpha) = a_{4ij}^{\xi} - \frac{(a_{4ij}^{\xi} - a_{3ij}^{\xi})\alpha}{h_{ij}^{\xi}}. \tag{7}
$$

With respect to the confidence interval at the $\alpha$ level ($[{}^lA_{ij}^L(\alpha), {}^rA_{ij}^L(\alpha)]$ and $[{}^lA_{ij}^U(\alpha), {}^rA_{ij}^U(\alpha)]$), the respective $\alpha$-level fuzzy interval $[{}^lA_{ij}^{\xi}(\alpha)_\alpha, {}^rA_{ij}^{\xi}(\alpha)_\alpha]$ for each $\xi \in \{L, U\}$ is defined as follows:

$$
\begin{aligned}
&[{}^lA_{ij}^{\xi}(\alpha)_\alpha, {}^rA_{ij}^{\xi}(\alpha)_\alpha](x_j) \\
&= \begin{cases} \alpha & \text{if } {}^lA_{ij}^{\xi}(\alpha) \le x_j \le {}^rA_{ij}^{\xi}(\alpha), \\ 0 & \text{otherwise.} \end{cases}
\end{aligned} \tag{8}
$$

Let the level 1 fuzzy number $\tilde{0}_1$ map onto the vertical axis at the origin. The signed distance of the crisp interval, $[{}^lA_{ij}^U(\alpha), {}^rA_{ij}^U(\alpha)]$, to $\tilde{0}_1$ is computed as follows:

$$
d\left(\left[{}^lA_{ij}^U(\alpha), {}^rA_{ij}^U(\alpha)\right], \tilde{0}_1\right) = \frac{1}{2}\left(d\left({}^lA_{ij}^U(\alpha), \tilde{0}_1\right) + d\left({}^rA_{ij}^U(\alpha), \tilde{0}_1\right)\right). \tag{9}
$$

The $\alpha$-cut, $[{}^lA_{ij}^U(\alpha), {}^rA_{ij}^U(\alpha)]$, is the one-one and onto mapping of the $\alpha$-level fuzzy interval, $[{}^lA_{ij}^U(\alpha)_\alpha, {}^rA_{ij}^U(\alpha)_\alpha]$. Thus, the signed distance from the $\alpha$-level fuzzy interval $[{}^lA_{ij}^U(\alpha)_\alpha, {}^rA_{ij}^U(\alpha)_\alpha]$ to $\tilde{0}_1$ is calculated as

$$
\begin{aligned}
&d\left(\left[{}^lA_{ij}^U(\alpha)_\alpha, {}^rA_{ij}^U(\alpha)_\alpha\right], \tilde{0}_1\right) \\
&= \frac{1}{2}\left(a_{1ij}^U + a_{4ij}^U + \left(a_{2ij}^U + a_{3ij}^U - a_{1ij}^U - a_{4ij}^U\right)\frac{\alpha}{h_{ij}^U}\right),
\end{aligned} \tag{10}
$$

where $d$ is a continuous function of $\alpha$ on $[0, h_{ij}^U]$. The signed distance from $A_{ij}^U$ to $\tilde{0}_1$ can be derived by the following definite integral:

$$
\begin{aligned}
&d\left(A_{ij}^U, \tilde{0}_1\right) \\
&= \frac{1}{h_{ij}^U}\int_0^{h_{ij}^U} \frac{1}{2}\left(a_{1ij}^U + a_{4ij}^U + \left(a_{2ij}^U + a_{3ij}^U - a_{1ij}^U - a_{4ij}^U\right)\frac{\alpha}{h_{ij}^U}\right)\mathrm{d}\alpha \\
&= \frac{1}{4}\left(a_{1ij}^U + a_{2ij}^U + a_{3ij}^U + a_{4ij}^U\right).
\end{aligned} \tag{11}
$$

Therefore, the signed distances from the crisp intervals, $[{}^lA_{ij}^U(\alpha), {}^lA_{ij}^L(\alpha)]$ and $[{}^rA_{ij}^L(\alpha), {}^rA_{ij}^U(\alpha)]$, to $\tilde{0}_1$ can be calculated separately as

$$
\begin{aligned}
&d\left(\left[{}^lA_{ij}^U(\alpha), {}^lA_{ij}^L(\alpha)\right], \tilde{0}_1\right) \\
&= \frac{1}{2}\left(d\left({}^lA_{ij}^U(\alpha), \tilde{0}_1\right) + d\left({}^lA_{ij}^L(\alpha), \tilde{0}_1\right)\right) \\
&= \frac{1}{2}\left(a_{1ij}^L + a_{1ij}^U + \left(a_{2ij}^L - a_{1ij}^L\right)\frac{\alpha}{h_{ij}^L} + \left(a_{2ij}^U - a_{1ij}^U\right)\frac{\alpha}{h_{ij}^U}\right),
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&d\left(\left[{}^rA_{ij}^L(\alpha), {}^rA_{ij}^U(\alpha)\right], \tilde{0}_1\right) \\
&= \frac{1}{2}\left(d\left({}^rA_{ij}^L(\alpha), \tilde{0}_1\right) + d\left({}^rA_{ij}^U(\alpha), \tilde{0}_1\right)\right) \\
&= \frac{1}{2}\left(a_{4ij}^L + a_{4ij}^U + \left(a_{3ij}^L - a_{4ij}^L\right)\frac{\alpha}{h_{ij}^L} + \left(a_{3ij}^U - a_{4ij}^U\right)\frac{\alpha}{h_{ij}^U}\right).
\end{aligned} \tag{13}
$$

In addition,

$$
\begin{aligned}
&d\left(\left[{}^lA_{ij}^U(\alpha), {}^lA_{ij}^L(\alpha)\right] \cup \left[{}^rA_{ij}^L(\alpha), {}^rA_{ij}^U(\alpha)\right], \tilde{0}_1\right) \\
&= \frac{1}{2}\left(d\left(\left[{}^lA_{ij}^U(\alpha), {}^lA_{ij}^L(\alpha)\right], \tilde{0}_1\right) + d\left(\left[{}^rA_{ij}^L(\alpha), {}^rA_{ij}^U(\alpha)\right], \tilde{0}_1\right)\right).
\end{aligned} \tag{14}
$$

The $\alpha$-cuts $[{}^lA_{ij}^U(\alpha), {}^lA_{ij}^L(\alpha)]$ and $[{}^rA_{ij}^L(\alpha), {}^rA_{ij}^U(\alpha)]$ are the one-one and onto mappings of the $\alpha$-level fuzzy intervals $[{}^lA_{ij}^U(\alpha)_\alpha, {}^lA_{ij}^L(\alpha)_\alpha]$ and $[{}^rA_{ij}^L(\alpha)_\alpha, {}^rA_{ij}^U(\alpha)_\alpha]$. The signed distances from the $\alpha$-level fuzzy intervals $[{}^{l}A_{ij}^{U}(\alpha)_{\alpha}, {}^{l}A_{ij}^{L}(\alpha)_{\alpha}] \cup [{}^{r}A_{ij}^{L}(\alpha)_{\alpha}, {}^{r}A_{ij}^{U}(\alpha)_{\alpha}]$ to $\tilde{0}_1$ are expressed as

$$d\left(\left[{}^{l}A_{ij}^{U}(\alpha)_{\alpha}, {}^{l}A_{ij}^{L}(\alpha)_{\alpha}\right] \cup \left[{}^{r}A_{ij}^{L}(\alpha)_{\alpha}, {}^{r}A_{ij}^{U}(\alpha)_{\alpha}\right], \tilde{0}_1\right) = \frac{1}{4}\left(a_{1ij}^{L} + a_{1ij}^{U} + a_{4ij}^{L} + a_{4ij}^{U} + \left(a_{2ij}^{L} + a_{3ij}^{L} - a_{1ij}^{L} - a_{4ij}^{L}\right)\frac{\alpha}{h_{ij}^{L}} + \left(a_{2ij}^{U} + a_{3ij}^{U} - a_{1ij}^{U} - a_{4ij}^{U}\right)\frac{\alpha}{h_{ij}^{U}}\right), \tag{15}$$

where $d$ is a continuous function of $\alpha$ on $[0, h_{ij}^{L}]$. When $0 \leq \alpha < h_{ij}^{L}$, the average value of $d$ can be obtained by the following definite integral:

$$\frac{1}{h_{ij}^{L}}\int_{0}^{h_{ij}^{L}} d\left(\left[{}^{l}A_{ij}^{U}(\alpha)_{\alpha}, {}^{l}A_{ij}^{L}(\alpha)_{\alpha}\right] \cup \left[{}^{r}A_{ij}^{L}(\alpha)_{\alpha}, {}^{r}A_{ij}^{U}(\alpha)_{\alpha}\right], \tilde{0}_1\right) \mathrm{d}\alpha = \frac{1}{8}\left(a_{1ij}^{L} + a_{2ij}^{L} + a_{3ij}^{L} + a_{4ij}^{L} + 2a_{1ij}^{U} + 2a_{4ij}^{U} + \left(a_{2ij}^{U} + a_{3ij}^{U} - a_{1ij}^{U} - a_{4ij}^{U}\right)\frac{h_{ij}^{L}}{h_{ij}^{U}}\right). \tag{16}$$

when $h_{ij}^{L} \leq \alpha < h_{ij}^{U}$, the average value of $d$ is as follows:

$$\frac{1}{h_{ij}^{U} - h_{ij}^{L}}\int_{h_{ij}^{L}}^{h_{ij}^{U}} d\left(\left[{}^{l}A_{ij}^{U}(\alpha)_{\alpha}, {}^{r}A_{ij}^{U}(\alpha)_{\alpha}\right], \tilde{0}_1\right) \mathrm{d}\alpha = \frac{1}{h_{ij}^{U} - h_{ij}^{L}}\int_{h_{ij}^{L}}^{h_{ij}^{U}} \frac{1}{2}\left(a_{1ij}^{U} + a_{4ij}^{U} + \left(a_{2ij}^{U} + a_{3ij}^{U} - a_{1ij}^{U} - a_{4ij}^{U}\right)\frac{\alpha}{h_{ij}^{U}}\right) \mathrm{d}\alpha = \frac{1}{4}\left(a_{1ij}^{U} + a_{2ij}^{U} + a_{3ij}^{U} + a_{4ij}^{U} + \left(a_{2ij}^{U} + a_{3ij}^{U} - a_{1ij}^{U} - a_{4ij}^{U}\right)\frac{h_{ij}^{L}}{h_{ij}^{U}}\right). \tag{17}$$

Therefore, the signed distance from $A_{ij}$ to $\tilde{0}_1$ (for $0 < h_{ij}^{L} \leq h_{ij}^{U} \leq 1$) is as follows:

$$d(A_{ij}, \tilde{0}_1) = \frac{1}{h_{ij}^{L}}\int_{0}^{h_{ij}^{L}} d\left(\left[{}^{l}A_{ij}^{U}(\alpha)_{\alpha}, {}^{l}A_{ij}^{L}(\alpha)_{\alpha}\right] \cup \left[{}^{r}A_{ij}^{L}(\alpha)_{\alpha}, {}^{r}A_{ij}^{U}(\alpha)_{\alpha}\right], \tilde{0}_1\right) \mathrm{d}\alpha + \frac{1}{h_{ij}^{U} - h_{ij}^{L}}\int_{h_{ij}^{L}}^{h_{ij}^{U}} d\left(\left[{}^{l}A_{ij}^{U}(\alpha)_{\alpha}, {}^{r}A_{ij}^{U}(\alpha)_{\alpha}\right], \tilde{0}_1\right) \mathrm{d}\alpha$$

$$= \frac{1}{8}\left(a_{1ij}^{L} + a_{2ij}^{L} + a_{3ij}^{L} + a_{4ij}^{L} + 4a_{1ij}^{U} + 2a_{2ij}^{U} + 2a_{3ij}^{U} + 4a_{4ij}^{U} + 3\left(a_{2ij}^{U} + a_{3ij}^{U} - a_{1ij}^{U} - a_{4ij}^{U}\right)\frac{h_{ij}^{L}}{h_{ij}^{U}}\right). \tag{18}$$

If $0 < h_{ij}^{L} = h_{ij}^{U} \leq 1$, then $d(A_{ij}, \tilde{0}_1) = \frac{1}{8}(a_{1ij}^{L} + a_{2ij}^{L} + a_{3ij}^{L} + a_{4ij}^{L} + a_{1ij}^{U} + 5a_{2ij}^{U} + 5a_{3ij}^{U} + a_{4ij}^{U})$.

Let $A_{ij}$ and $A_{i'j'}$ be two IT2TrFN ratings. Because the signed distances $d(A_{ij}, \tilde{0}_1)$ and $d(A_{i'j'}, \tilde{0}_1)$ are real numbers, they satisfy the criteria of linear ordering. That is, one of the following three conditions must hold: $d(A_{ij}, \tilde{0}_1) > d(A_{i'j'}, \tilde{0}_1)$, $d(A_{ij}, \tilde{0}_1) = d(A_{i'j'}, \tilde{0}_1)$, or $d(A_{ij}, \tilde{0}_1) < d(A_{i'j'}, \tilde{0}_1)$. Subsequently, the signed distance based on IT2TrFNs satisfies the law of trichotomy. A comparison of the IT2TrFN ratings can be made via the signed distance from the IT2TrFN value to $\tilde{0}_1$.

## 3 Interval type-2 fuzzy PROMETHEE

The basic principle of PROMETHEE is based on a pairwise comparison of alternatives for each recognized criterion (Behzadian et al. 2010). Alternatives are evaluated according to different criteria that have to be maximized (i.e., benefit criteria) or minimized (i.e., cost criteria). In PROMETHEE I, partial rankings are obtained by calculating the positive and negative outranking flows; both flows do not usually yield the same rankings (Vinodh and Jeya Girubha 2012). Because the decision maker always desires full ranking, PROMETHEE II is appropriately employed for the evaluation (Brans and Vincke 1985). This study develops interval type-2 fuzzy PROMETHEE I and II methods to address MCDA problems. The proposed methods begin with the formulation of the evaluative rating $A_{ij}$ and the importance weight $W_j$ within the IT2TrFN environment.

### 3.1 Signed distance-based generalized criteria

For any two alternatives $z_\rho$ and $z_\beta$ ($z_\rho, z_\beta \in Z$) with respect to each criterion $x_j \in X$, the pairwise comparison of the evaluative ratings $A_{\rho j}$ and $A_{\beta j}$ can be indicated by preference function $h(A_{\rho j}, A_{\beta j})$. Recall that $A_{\rho j} = [A_{\rho j}^{L}, A_{\rho j}^{U}] = [(a_{1\rho j}^{L}, a_{2\rho j}^{L}, a_{3\rho j}^{L}, a_{4\rho j}^{L}; h_{\rho j}^{L}), (a_{1\rho j}^{U}, a_{2\rho j}^{U}, a_{3\rho j}^{U}, a_{4\rho j}^{U}; h_{\rho j}^{U})]$ and $A_{\beta j} = [A_{\beta j}^{L}, A_{\beta j}^{U}] = [(a_{1\beta j}^{L}, a_{2\beta j}^{L}, a_{3\beta j}^{L}, a_{4\beta j}^{L}; h_{\beta j}^{L}), (a_{1\beta j}^{U}, a_{2\beta j}^{U}, a_{3\beta j}^{U}, a_{4\beta j}^{U}; h_{\beta j}^{U})]$. Let the preference function $h(A_{\rho j}, A_{\beta j})$ denote the intensity of the preference of $A_{\rho j}$ over $A_{\beta j}$. The preference function $h(A_{\rho j}, A_{\beta j})$ has the following meanings:

(i) $h(A_{\rho j}, A_{\beta j}) = 0$ indicates an indifference between $A_{\rho j}$ and $A_{\beta j}$ or no preference of $A_{\rho j}$ over $A_{\beta j}$;

(ii) $h(A_{\rho j}, A_{\beta j}) \sim 0$ indicates a weak preference of $A_{\rho j}$ over $A_{\beta j}$;
(iii) $h(A_{\rho j}, A_{\beta j}) \sim 1$ indicates a strong preference of $A_{\rho j}$ over $A_{\beta j}$;
(iv) $h(A_{\rho j}, A_{\beta j}) = 1$ indicates a strict preference of $A_{\rho j}$ over $A_{\beta j}$.

Because the signed distances of $d(A_{\rho j}, \tilde{0}_1)$ and $d(A_{\beta j}, \tilde{0}_1)$ can be used to order $A_{\rho j}$ and $A_{\beta j}$, the preference function $h(A_{\rho j}, A_{\beta j})$ can be defined as a function of the difference between $d(A_{\rho j}, \tilde{0}_1)$ and $d(A_{\beta j}, \tilde{0}_1)$. Let

$$D = \begin{cases} d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1) & \text{if } x_j \in X_b, \\ d(A_{\beta j}, \tilde{0}_1) - d(A_{\rho j}, \tilde{0}_1) & \text{if } x_j \in X_c. \end{cases} \tag{19}$$

To better define the indifference area, we consider a function $H(D)$ that is directly related to the preference function $h$ as follows:

$$H(D) = \begin{cases} h(A_{\rho j}, A_{\beta j}) & \text{if } D \geq 0, \\ h(A_{\beta j}, A_{\rho j}) & \text{if } D \leq 0. \end{cases} \tag{20}$$

Classical PROMETHEE suggests several types of preference functions to express the importance of the relative difference between alternatives for a certain criterion and weights to indicate the relative importance of the criterion (Vinodh and Jeya Girubha 2012). To facilitate the selection of a specific preference function, Brans and Vincke (1985) proposed six basic types of generalized criteria: the (1) usual criterion, (2) U-shaped criterion, (3) V-shaped criterion, (4) level criterion, (5) V-shaped with indifference criterion, and (6) Gaussian criterion. For each generalized criterion, the value of an indifference threshold, $q$, the value of a strict preference threshold, $p$, and the standard deviation of a normal distribution, $\alpha$, have to be fixed values (Brans and Mareschal 1992). In each case, these parameters convey a clear significance for the decision maker (Behzadian et al. 2010). In this paper, we consider the following six types of signed distance-based generalized criteria:

(i) Type I: Signed distance-based usual criterion:

$$H(D) = \begin{cases} 0 & \text{if } d(A_{\rho j}, \tilde{0}_1) = d(A_{\beta j}, \tilde{0}_1), \\ 1 & \text{otherwise.} \end{cases} \tag{21}$$

In this case, there is indifference between $A_{\rho j}$ and $A_{\beta j}$ if and only if $d(A_{\rho j}, \tilde{0}_1) = d(A_{\beta j}, \tilde{0}_1)$. If the two signed distances are different, the decision maker has a strict preference for the alternative that has the larger signed distance. Note that no parameter has to be defined in this generalized criterion.

(ii) Type II: Signed distance-based U-shaped criterion:

$$H(D) = \begin{cases} 0 & \text{if } \left| d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1) \right| \leq q, \\ 1 & \text{otherwise.} \end{cases} \tag{22}$$

The two alternatives $A_\rho$ and $A_\beta$ with respect to $x_j$ are indifferent to the decision maker as long as the absolute value of the difference between their signed distances to $\tilde{0}_1$ does not exceed the indifference threshold $q$. If this is not the case, there is strict preference. The decision maker has to designate the value of $q$. The indifference threshold $q$ is the largest value of $|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|$ below which the decision maker considers the corresponding alternatives indifferent.

(iii) Type III: Signed distance-based V-shaped criterion:

$$H(D) = \begin{cases} \frac{|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|}{p} & \text{if } |d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)| \leq p, \\ 1 & \text{otherwise.} \end{cases} \tag{23}$$

As long as $|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|$ is smaller than the value of $p$, the preference of the decision maker increases linearly with $|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|$. If $|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|$ becomes larger than $p$, there is a strict preference situation. The preference threshold $p$ is the smallest value of $|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|$ above which there is a strict preference.

(iv) Type IV: Signed distance-based level criterion:

$$H(D) = \begin{cases} 0 & \text{if } |d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)| \leq q, \\ \frac{1}{2} & \text{if } q < |d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)| \leq p, \\ 1 & \text{otherwise.} \end{cases} \tag{24}$$

In this case, the indifference threshold $q$ and the preference threshold $p$ are simultaneously employed. If $|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)|$ lies between $q$ and $p$, there is a weak preference situation ($H(D) = 1/2$). The decision maker has to define the two thresholds in this signed distance-based level criterion.

(v) Type V: Signed distance-based V-shaped with indifference criterion:

$$H(D) = \begin{cases} 0 & \text{if } |d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)| \leq q, \\ \frac{|d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)| - q}{p - q} & \text{if } q < |d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)| \leq p, \\ 1 & \text{otherwise.} \end{cases} \tag{25}$$

In this case, the decision maker considers that his/her preference increases linearly from indifference to strict preference in the area between the two thresholds $q$ and $p$. Two parameters have to be defined.

(vi) Type VI: Signed distance-based Gaussian criterion:

$$H(D) = 1 - e^{-\frac{(d(A_{\rho j},\tilde{0}_1) - d(A_{\beta j},\tilde{0}_1))^2}{2\sigma^2}}. \tag{26}$$

The definition of the Gaussian criterion has no discontinuities and leads to guaranteed stability of the results. The Gaussian criterion only requires establishing $\alpha$, which is facilitated through experience with normal distributions in statistics. The parameter $\alpha$ is directly connected with the standard deviation of a normal distribution.

## 3.2 Signed distance-based comprehensive preference index

Consider that the criterion importance defined on $X$ is expressed as $W = \{\langle x_j, W_j\rangle | x_j \in X, j = 1, 2, \ldots, n\}$, where $W_j = [(w_{1j}^L, w_{2j}^L, w_{3j}^L, w_{4j}^L; h_j^L), (w_{1j}^U, w_{2j}^U, w_{3j}^U, w_{4j}^U; h_j^U)]$. In addition, $h(A_{\rho j}, A_{\beta j}) = H(D)$ when $D \geq 0$; otherwise, $h(A_{\rho j}, A_{\beta j}) = 0$. Let $\hbar$ denote the signed distance-based comprehensive preference index. For any two alternatives $z_\rho$ and $z_\beta$ ($z_\rho, z_\beta \in Z$), the index $\hbar$ is defined as the weighted average of the preference functions by the following:

$$\hbar(z_\rho, z_\beta) = \left(\overset{n}{\underset{j=1}{\oplus}} h(A_{\rho j}, A_{\beta j}) \cdot W_j\right) \varnothing \left(\overset{n}{\underset{j=1}{\oplus}} W_j\right). \tag{27}$$

Specifically, by applying the arithmetic operations of IT2TrFN values listed in the Appendix, we obtain

$$\begin{aligned}
&\hbar(z_\rho, z_\beta)\\
&= \left[\left(\frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{1j}^L}{\sum_{j=1}^n w_{4j}^L}, \frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{2j}^L}{\sum_{j=1}^n w_{3j}^L},\right.\right.\\
&\left.\frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{3j}^L}{\sum_{j=1}^n w_{2j}^L}, \frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{4j}^L}{\sum_{j=1}^n w_{1j}^L}; \min_{j=1}^n h_j^L\right),\\
&\left(\frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{1j}^U}{\sum_{j=1}^n w_{4j}^U}, \frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{2j}^U}{\sum_{j=1}^n w_{3j}^U},\right.\\
&\left.\left.\frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{3j}^U}{\sum_{j=1}^n w_{2j}^U}, \frac{\sum_{j=1}^n h(A_{\rho j}, A_{\beta j}) \cdot w_{4j}^U}{\sum_{j=1}^n w_{1j}^U}; \min_{j=1}^n h_j^U\right)\right].
\end{aligned}$$

For brevity, we define $\hbar_{1\rho\beta}^\xi = \sum h(A_{\rho j}, A_{\beta j}) \cdot w_{1j}^\xi / \sum w_{4j}^\xi$, $\hbar_{2\rho\beta}^\xi = \sum h(A_{\rho j}, A_{\beta j}) \cdot w_{2j}^\xi / \sum w_{3j}^\xi$, $\hbar_{3\rho\beta}^\xi = \sum h(A_{\rho j}, A_{\beta j}) \cdot w_{3j}^\xi / \sum w_{2j}^\xi$, $\hbar_{4\rho\beta}^\xi = \sum h(A_{\rho j}, A_{\beta j}) \cdot w_{4j}^\xi / \sum w_{1j}^\xi$, and $h_{\rho\beta}^\xi = \min_{j=1}^n h_j^\xi$ for $\xi \in \{L, U\}$. The signed distance-based comprehensive preference index $\hbar(z_\rho, z_\beta)$ of the alternatives $z_\rho$ and $z_\beta$ can then be expressed as

$$\begin{aligned}
\hbar(z_\rho, z_\beta) = &\left[\left(\hbar_{1\rho\beta}^L, \hbar_{2\rho\beta}^L, \hbar_{3\rho\beta}^L, \hbar_{4\rho\beta}^L; h_{\rho\beta}^L\right),\right.\\
&\left.\left(\hbar_{1\rho\beta}^U, \hbar_{2\rho\beta}^U, \hbar_{3\rho\beta}^U, \hbar_{4\rho\beta}^U; h_{\rho\beta}^U\right)\right],
\end{aligned} \tag{28}$$

where $0 \leq \hbar_{1\rho\beta}^L \leq \hbar_{2\rho\beta}^L \leq \hbar_{3\rho\beta}^L \leq \hbar_{4\rho\beta}^L$, $0 \leq \hbar_{1\rho\beta}^U \leq \hbar_{2\rho\beta}^U \leq \hbar_{3\rho\beta}^U \leq \hbar_{4\rho\beta}^U$, $0 \leq h_{\rho\beta}^L \leq h_{\rho\beta}^U \leq 1$, $\hbar_{1\rho\beta}^U \leq \hbar_{1\rho\beta}^L$, and $\hbar_{4\rho\beta}^L \leq \hbar_{4\rho\beta}^U$.

When simultaneously considering all criteria, $\hbar(z_\rho, z_\beta)$ represents the intensity of preference of alternative $z_\rho$ over alternative $z_\beta$ by the decision maker. When $\hbar(z_\rho, z_\beta) \approx$ [(0, 0, 0, 0; 1), (0, 0, 0, 0; 1)], a weak preference for $z_\rho$ over $z_\beta$ is implied based on all criteria. When $\hbar(z_\rho, z_\beta) \approx$ [(1, 1, 1, 1; 1), (1, 1, 1, 1; 1)], a strong preference for $z_\rho$ over $z_\beta$ is implied based on all criteria.

## 3.3 Interval type-2 fuzzy PROMETHEE I and II rankings

The signed distance-based comprehensive preference index determines a valued outranking relation on the set $Z$ of alternatives. This relation can be represented as a valued outranking graph in which the nodes are the alternatives of $Z$. Between the two nodes (i.e., alternatives) $z_\rho$ and $z_\beta$, there are two arcs with IT2TrFN values $\hbar(z_\rho, z_\beta)$ and $\hbar(z_\beta, z_\rho)$. To evaluate the alternatives in $Z$ by using the outranking relation, we consider the following flows:

(i) The leaving flow:

$$\begin{aligned}
\Phi^+(z_i) &= \overset{m}{\underset{\beta=1,\beta\neq i}{\oplus}} \hbar(z_i, z_\beta)\\
&= \left[\left(\sum_{\beta=1,\beta\neq i}^m \hbar_{1i\beta}^L, \sum_{\beta=1,\beta\neq i}^m \hbar_{2i\beta}^L, \sum_{\beta=1,\beta\neq i}^m \hbar_{3i\beta}^L,\right.\right.\\
&\qquad\left.\sum_{\beta=1,\beta\neq i}^m \hbar_{4i\beta}^L; \min_{\beta=1,\beta\neq i}^m h_{i\beta}^L\right),\\
&\quad\left(\sum_{\beta=1,\beta\neq i}^m \hbar_{1i\beta}^U, \sum_{\beta=1,\beta\neq i}^m \hbar_{2i\beta}^U, \sum_{\beta=1,\beta\neq i}^m \hbar_{3i\beta}^U,\right.\\
&\quad\left.\left.\sum_{\beta=1,\beta\neq i}^m \hbar_{4i\beta}^U; \min_{\beta=1,\beta\neq i}^m h_{i\beta}^U\right)\right].
\end{aligned} \tag{29}$$

The leaving flow is the sum of the IT2TrFN values of the arcs leaving node $z_i$, which provides a measure of the outranking character of $z_i$ (i.e., how $z_i$ outranks all other alternatives of $Z$).

(ii) The entering flow:

$$\begin{aligned}
\Phi^-(z_i) &= \overset{m}{\underset{\rho=1,\rho\neq i}{\oplus}} \hbar(z_\rho, z_i)\\
&= \left[\left(\sum_{\rho=1,\rho\neq i}^m \hbar_{1\rho i}^L, \sum_{\rho=1,\rho\neq i}^m \hbar_{2\rho i}^L, \sum_{\rho=1,\rho\neq i}^m \hbar_{3\rho i}^L,\right.\right.\\
&\qquad\left.\sum_{\rho=1,\rho\neq i}^m \hbar_{4\rho i}^L; \min_{\rho=1,\rho\neq i}^m h_{\rho i}^L\right),
\end{aligned}$$

$$\left(\sum_{\rho=1,\rho\neq i}^{m} \hbar_{1\rho i}^{U}, \sum_{\rho=1,\rho\neq i}^{m} \hbar_{2\rho i}^{U}, \sum_{\rho=1,\rho\neq i}^{m} \hbar_{3\rho i}^{U}, \sum_{\rho=1,\rho\neq i}^{m} \hbar_{4\rho i}^{U}; \min_{\rho=1,\rho\neq i}^{m} h_{\rho i}^{U}\right)\Bigg]. \tag{30}$$

The entering flow is the sum of the IT2TrFN values of the arcs entering node $z_i$, which provides a measure of the outranked character of $z_i$ (i.e., how $z_i$ is dominated by all other alternatives of $Z$).

(iii) The net flow:

$$\Phi(z_i) = (\Phi^{+}(z_i))\Theta(\Phi^{-}(z_i))$$
$$= \Bigg[\Bigg(\sum_{\beta=1,\beta\neq i}^{m} \hbar_{1i\beta}^{L} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{4\rho i}^{L}, \sum_{\beta=1,\beta\neq i}^{m} \hbar_{2i\beta}^{L} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{3\rho i}^{L}, \sum_{\beta=1,\beta\neq i}^{m} \hbar_{3i\beta}^{L} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{2\rho i}^{L}, \sum_{\beta=1,\beta\neq i}^{m} \hbar_{4i\beta}^{L} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{1\rho i}^{L}; \min\left\{\min_{\beta=1,\beta\neq i}^{m} h_{i\beta}^{L}, \min_{\rho=1,\rho\neq i}^{m} h_{\rho i}^{L}\right\}\Bigg),$$
$$\Bigg(\sum_{\beta=1,\beta\neq i}^{m} \hbar_{1i\beta}^{U} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{4\rho i}^{U}, \sum_{\beta=1,\beta\neq i}^{m} \hbar_{2i\beta}^{U} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{3\rho i}^{U}, \sum_{\beta=1,\beta\neq i}^{m} \hbar_{3i\beta}^{U} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{2\rho i}^{U}, \sum_{\beta=1,\beta\neq i}^{m} \hbar_{4i\beta}^{U} - \sum_{\rho=1,\rho\neq i}^{m} \hbar_{1\rho i}^{U}; \min\left\{\min_{\beta=1,\beta\neq i}^{m} h_{i\beta}^{U}, \min_{\rho=1,\rho\neq i}^{m} h_{\rho i}^{U}\right\}\Bigg)\Bigg]. \tag{31}$$

the case of the interval type-2 fuzzy PROMETHEE I method, we employ the rationale that the higher the leaving flow and the lower the entering flow, the better the alternative. By employing the signed distances $d(\Phi^{+}(z_i), \tilde{0}_1)$ and $d(\Phi^{-}(z_i), \tilde{0}_1)$ for each alternative $z_i \in Z$, the leaving and entering flows, respectively, induce the following procedures:

$$\begin{cases} z_i \succ^{+} z_j & \text{if and only if } d(\Phi^{+}(z_i), \tilde{0}_1) > d(\Phi^{+}(z_j), \tilde{0}_1), \\ z_i \sim^{+} z_j & \text{if and only if } d(\Phi^{+}(z_i), \tilde{0}_1) = d(\Phi^{+}(z_j), \tilde{0}_1); \end{cases} \tag{32}$$

$$\begin{cases} z_i \succ^{-} z_j & \text{if and only if } d(\Phi^{-}(z_i), \tilde{0}_1) < d(\Phi^{-}(z_j), \tilde{0}_1), \\ z_i \sim^{-} z_j & \text{if and only if } d(\Phi^{-}(z_i), \tilde{0}_1) = d(\Phi^{-}(z_j), \tilde{0}_1). \end{cases} \tag{33}$$

The interval type-2 fuzzy PROMETHEE I partial preorder $(\succ^{\mathrm{I}}, \sim^{\mathrm{I}}, \mathbb{R})$ is then obtained by considering the intersection of these two preorders:

$$\begin{cases} z_i \succ^{\mathrm{I}} z_j \ (z_i \text{ outranks } z_j) & \text{if } z_i \succ^{+} z_j \text{ and } z_i \succ^{-} z_j, \\ & \text{or } z_i \succ^{+} z_j \text{ and } z_i \sim^{-} z_j, \\ & \text{or } z_i \sim^{+} z_j \text{ and } z_i \succ^{-} z_j; \\ z_i \sim^{\mathrm{I}} z_j \ (z_i \text{ is indifferent to } z_j) & \text{if and only if } z_i \sim^{+} z_j \text{ and } z_i \sim^{-} z_j; \\ z_i \,\mathbb{R}\, z_j \ (z_i \text{ and } z_j \text{ are incomparable}) & \text{otherwise.} \end{cases} \tag{34}$$

Note that only confirmed outranking relations are given by the partial preorder. Some alternatives may remain incomparable by using the proposed interval type-2 fuzzy PROMETHEE I method.

In the case of the interval type-2 fuzzy PROMETHEE II method, we compute the signed distance $d(\Phi(z_i), \tilde{0}_1)$ for each alternative $z_i \in Z$. Then, a complete preorder $(\succ^{\mathrm{II}}, \sim^{\mathrm{II}})$ is induced by the net flow as follows:

$$\begin{cases} z_i \succ^{\mathrm{II}} z_j \ (z_i \text{ outranks } z_j) & \text{if and only if } d(\Phi(z_i), \tilde{0}_1) > d(\Phi(z_j), \tilde{0}_1), \\ z_i \sim^{\mathrm{II}} z_j \ (z_i \text{ is indifferent to } z_j) & \text{if and only if } d(\Phi(z_i), \tilde{0}_1) = d(\Phi(z_j), \tilde{0}_1). \end{cases} \tag{35}$$

To avoid any incomparability, use of the interval type-2 fuzzy PROMETHEE II method can produce the complete preorder on $Z$. Thus, it is easier for the decision maker to resolve the decision-making problem by using the complete preorder.

### 3.4 The proposed algorithm

The procedure for implementing the proposed interval type-2 fuzzy PROMETHEE methods is started to determine the difference between signed distances based on pairwise comparisons of evaluative ratings. This step is followed by using a relevant preference function for each criterion, calculating signed distance-based comprehensive preference indices, and determining leaving and entering flows for each alternative. The procedure comes to an end with the determination of partial rankings in the interval type-2 fuzzy PROMETHEE I method or the calculation of net flows for each alternative and complete rankings in the interval type-2 fuzzy PROMETHEE II method.

Based on IT2TrFNs, the interval type-2 fuzzy PROMETHEE I method for solving an MCDA problem is summarized in the following steps:

Step 1: Formulate an MCDA problem. Specify the alternative set $Z = \{z_1, z_2, \ldots, z_m\}$ and the criterion set $X = \{x_1, x_2, \ldots, x_n\}$ that is divided into $X_b$ and $X_c$.

Step 2: Designate the appropriate type of generalized criteria for each $x_j \in X$. Request that the decision maker specify the corresponding parameters (the indifference threshold $q$, the preference threshold $p$, or the standard deviation $\alpha$ of a normal distribution).

Step 3: Select the appropriate linguistic variables or other data collection tools to establish the IT2TrFN rating $A_{ij}$ in (1) for alternative $z_i \in Z$ with respect to criterion $x_j \in X$ and the importance weight $W_j$ in (3) for criterion $x_j \in X$, which are provided by the decision maker.

Step 4: By applying (18), calculate the signed distance $d(A_{ij}, \tilde{0}_1)$ from $A_{ij}$ to $\tilde{0}_1$ for each alternative $z_i$ with respect to the criterion $x_j$.

Step 5: Use the appropriate types of signed distance-based generalized criteria in (21)–(26) to acquire the preference function $h(A_{\rho j}, A_{\beta j})$ for each pairwise comparison of alternatives $z_\rho, z_\beta \in Z$ with respect to $x_j \in X$.

Step 6: Consider the criterion importance $W_j$ of each criterion $x_j \in X$ to compute the signed distance-based comprehensive preference index $\hbar(z_\rho, z_\beta)$ for each pair of $(z_\rho, z_\beta)$ using (27).

Step 7: Apply (29) and (30) to obtain the leaving flow $\Phi^+(z_i)$ and entering flow $\Phi^-(z_i)$ for alternative $z_i \in Z$. Then, compute the signed distances $d(\Phi^+(z_i), \tilde{0}_1)$ and $d(\Phi^-(z_i), \tilde{0}_1)$ for $z_i \in Z$.

Step 8: Follow the procedures in (32) and (33) to determine the partial preorder for the set $Z$ of alternatives by using $(\succ^{\mathrm{I}}, \sim^{\mathrm{I}}, \mathbb{R})$ in (34).

The interval type-2 fuzzy PROMETHEE II method, which is based on IT2TrFNs for solving an MCDA problem, is summarized in the following steps:

Steps 1–6: See Steps 1–6 of the interval type-2 fuzzy PROMETHEE I method.

Step 7: Apply (31) to obtain the net flow $\Phi(z_i)$ for alternative $z_i \in Z$. Then, compute the signed distance $d(\Phi(z_i), \tilde{0}_1)$ for $z_i \in Z$.

Step 8: Determine the complete preorder for the set $Z$ of alternatives by using $(\succ^{\mathrm{II}}, \sim^{\mathrm{II}})$ in (35).

## 4 Case illustration

In this section, we examine a real-world landfill siting problem in KS City, adapted from Chen (2011b), and discuss how the proposed interval type-2 fuzzy PROMETHEE I and II outranking methods are implemented in practice. Chen (2011b) presented an integrated approach that combines the objective and subjective importance of decision criteria and assessed the criterion importance in the problem of landfill site selection. This paper utilizes data similar to the data from the landfill siting problem; however, we apply the proposed PROMETHEE I and PROMETHEE II methods to determine the partial and complete rankings, respectively, of candidate locations.

### 4.1 Illustration of the algorithm

The proposed interval type-2 fuzzy PROMETHEE I and II outranking methods were applied to solve the problem of landfill site selection introduced by Chen (2011b), and the computational procedure is summarized below.

In Step 1, there are four available landfill sites; the set of all candidate locations is denoted by $Z = \{z_1, z_2, z_3, z_4\}$. The seven evaluation criteria for landfill site selection are considered, including transportation convenience ($x_1$), terrain suitability ($x_2$), community equity ($x_3$), environmental impact ($x_4$), ecological impact ($x_5$), construction cost ($x_6$), and historic impact ($x_7$). The criteria $x_1$, $x_2$, and $x_3$ denote benefit criteria, whereas all others denote cost criteria. The set of evaluative criteria is denoted by $X = \{x_1, x_2, x_3, \ldots, x_7\}$ with $X_b = \{x_1, x_2, x_3\}$ and $X_c = \{x_4, x_5, x_6, x_7\}$.

Table 2 indicates the types of generalized criteria and the corresponding parameters for each criterion in Step 2. The table also illustrates that if the benefit criteria have to be maximized, the cost criteria also have to be minimized. For convenience, the decision maker can use linguistic variables to describe ratings of alternatives with respect to various criteria, and then these linguistic variables can be converted into IT2TrFNs. This paper adopts Chen's (2011b) nine-point rating scales to establish IT2TrFN ratings of the alternatives and importance weights of the criteria. Table 2 depicts the linguistic evaluations of the four candidate locations.

In Step 3, we convert linguistic variables to IT2TrFNs according to Chen's (2011b) approach. Table 3 shows the importance weights and the IT2TrFN ratings in the

**Table 2** Criterion characteristics and linguistic evaluations of the candidate locations

| Evaluative criteria | Direction | Importance weight | Candidate locations | | | | Type of the preference function | Parameters |
|---|---|---|---|---|---|---|---|---|
| | | | $z_1$ | $z_2$ | $z_3$ | $z_4$ | | |
| $x_1$ (transportation convenience) | Max | H | L | AL | AH | H | V-shaped with indifference | $q = 0.1,\ p = 0.5$ |
| $x_2$ (terrain suitability) | Max | MH | AH | VH | AL | L | Usual | – |
| $x_3$ (community equity) | Max | H | AL | H | MH | H | Level | $q = 0.1,\ p = 0.5$ |
| $x_4$ (environmental impact) | Min | MH | L | M | AH | VH | U-shaped | $q = 0.1$ |
| $x_5$ (ecological impact) | Min | MH | L | ML | AH | MH | Gaussian | $\sigma = 0.4$ |
| $x_6$ (construction cost) | Min | M | AH | ML | M | M | Gaussian | $\sigma = 0.4$ |
| $x_7$ (historic impact) | Min | ML | AH | L | VH | H | V-shaped | $p = 0.5$ |

landfill siting problem. In addition, this table lists the computation results of the signed distance $d(A_{ij}, \tilde{0}_1)$ from $A_{ij}$ to $\tilde{0}_1$ according to Step 4. For example, consider $A_{24}$ $= [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1)]$. The signed distance $d(A_{24}, \tilde{0}_1)$ is computed using (18) as follows:

$$
\begin{aligned}
d(\bar{A}_{24}, \tilde{0}_1) = \frac{1}{8}\Big(&0.4025 + 0.4525 + 0.5375 + 0.5675 + 4 \\
&\times\ 0.3200 + 2 \times 0.4100 + 2 \times 0.5800 + 4 \times 0.6500 \\
&+ 3(0.4100 + 0.5800 - 0.3200 - 0.6500)\frac{0.8}{1}\Big) = 0.9835.
\end{aligned}
$$

In Step 5, we calculate the preference function $h(A_{\rho j}, A_{\beta j})$ for each pairwise comparison of alternatives $z_\rho, z_\beta \in Z$ with respect to $x_j \in X$. Consider $h(A_{1j}, A_{2j})$ for example. As indicated in Table 2, the benefit criterion $x_1$ belongs to Type V (i.e., the signed distance-based V-shaped with indifference criterion), and the given parameters are $q = 0.1$ and $p = 0.5$. Applying (19), we obtain $D = d(A_{11}, \tilde{0}_1) - d(A_{21}, \tilde{0}_1) = 0.2768 - 0.0000 = 0.2768$ because $x_1 \in X_b$; then, it follows that $h(A_{11}, A_{21}) = H(D)$ because $D(= 0.2768) \geq 0$ using (20). Because $0.1 < |d(A_{11}, \tilde{0}_1) - d(A_{21}, \tilde{0}_1)| \leq 0.5$, the preference function $h(A_{11}, A_{21})$ is derived using (25) as follows:

$$
h(A_{11}, A_{21}) = H(D) = \frac{|0.2768 - 0.0000| - 0.1}{0.5 - 0.1} = 0.4420.
$$

For the benefit criterion $x_2$, which belongs to Type I (i.e., the signed distance-based usual criterion), we have $D = d(A_{12}, \tilde{0}_1) - d(A_{22}, \tilde{0}_1) = 2.0000 - 1.9647 = 0.0353 \geq 0$. According to (21), $h(A_{12}, A_{22}) = H(D) = 1$ because $d(A_{12}, \tilde{0}_1) \neq d(A_{22}, \tilde{0}_1)$.

Next, for the cost criterion $x_4$, which belongs to Type II (i.e., the signed distance-based U-shaped criterion) with the parameter $q = 0.1$, we obtain $D = d(A_{24}, \tilde{0}_1) - d(A_{14}, \tilde{0}_1) = 0.9835 - 0.2768 = 0.7067 \geq 0$ because $x_4 \in X_c$. According to (22), we have $h(A_{14}, A_{24}) = H(D) = 1$ because $|d(A_{14}, \tilde{0}_1) - d(A_{24}, \tilde{0}_1)| > 0.5$. The cost criterion $x_5$ belongs to Type VI (i.e., the signed distance-based Gaussian criterion) with the parameter $\alpha = 0.4$. Because $d(A_{25}, \tilde{0}_1) - d(A_{15}, \tilde{0}_1) = 0.5833 - 0.2768 = 0.3065 \geq 0$, the preference function $h(A_{15}, A_{25})$ is calculated using (26) as follows:

$$
\begin{aligned}
h(A_{15}, A_{25}) &= H(D) = 1 - e^{-\frac{(0.2768 - 0.5833)^2}{2 \times 0.4^2}} \\
&= 0.2544.
\end{aligned}
$$

For the benefit criterion $x_3$, we obtain $h(A_{13}, A_{23}) = 0$ because $d(A_{13}, \tilde{0}_1) - d(A_{23}, \tilde{0}_1) = 0.0000 - 1.6968 = -1.6968 < 0$. For the cost criteria $x_6$ and $x_7$, we have $d(A_{26}, \tilde{0}_1) - d(A_{16}, \tilde{0}_1) = 0.5833 - 2.0000 = -1.4167 < 0$ and $d(A_{27}, \tilde{0}_1) - d(A_{17}, \tilde{0}_1) = 0.2768 - 2.0000 = -1.7232 < 0$; thus, $h(A_{16}, A_{26}) = h(A_{17}, A_{27}) = 0$. The computation results of the preference function $h(A_{\rho j}, A_{\beta j})$ are shown in Table 4.

In Step 6, the computation results of the signed distance-based comprehensive preference index $\hbar(z_\rho, z_\beta)$ for each pair of $(z_\rho, z_\beta)$ are also depicted in Table 4. For example, $\hbar(z_1, z_2) = [(\hbar^L_{112}, \hbar^L_{212}, \hbar^L_{312}, \hbar^L_{412}; h^L_{12}), (\hbar^U_{112}, \hbar^U_{212}, \hbar^U_{312}, \hbar^U_{412}; h^U_{12})]$, where

$$
\begin{aligned}
\hbar^L_{112} &= \frac{\sum_{j=1}^{7} h(A_{1j}, A_{2j}) \cdot w^L_{1j}}{\sum_{j=1}^{7} w^L_{4j}} \\
&= \frac{0.442 \times 0.7825 + 1 \times 0.65 + 0 \times 0.7825 + 1 \times 0.65 + 0.2544 \times 0.65 + 0 \times 0.4025 + 0 \times 0.2325}{0.9075 + 0.79 + 0.9075 + 0.79 + 0.79 + 0.5675 + 0.3575} \\
&= 0.3544.
\end{aligned}
$$

**Table 3** The importance weights and the IT2TrFN ratings.

| | |
|---|---|
| The importance weight $W_j$ of criterion $x_j \in X$ | |
| $W_1 = [(0.7825, 0.8150, 0.8850, 0.9075; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1)]$ | |
| $W_2 = [(0.6500, 0.6725, 0.7575, 0.7900; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1)]$ | |
| $W_3 = [(0.7825, 0.8150, 0.8850, 0.9075; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1)]$ | |
| $W_4 = [(0.6500, 0.6725, 0.7575, 0.7900; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1)]$ | |
| $W_5 = [(0.6500, 0.6725, 0.7575, 0.7900; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1)]$ | |
| $W_6 = [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1)]$ | |
| $W_7 = [(0.2325, 0.2550, 0.3250, 0.3575; 0.8), (0.1700, 0.2200, 0.3600, 0.4200; 1)]$ | |
| The IT2TrFN rating $A_{ij}$ of alternative $z_i \in Z$ with respect to criterion $x_j \in X$ | $d(A_{ij}, \tilde{0}_1)$ |
| $A_{11} = [(0.0875, 0.1200, 0.1600, 0.1825; 0.8), (0.0400, 0.1000, 0.1800, 0.2300; 1)]$ | 0.2768 |
| $A_{12} = [(1.0000, 1.0000, 1.0000, 1.0000; 1), (1.0000, 1.0000, 1.0000, 1.0000; 1)]$ | 2.0000 |
| $A_{13} = [(0.0000, 0.0000, 0.0000, 0.0000; 1), (0.0000, 0.0000, 0.0000, 0.0000; 1)]$ | 0.0000 |
| $A_{14} = [(0.0875, 0.1200, 0.1600, 0.1825; 0.8), (0.0400, 0.1000, 0.1800, 0.2300; 1)]$ | 0.2768 |
| $A_{15} = [(0.0875, 0.1200, 0.1600, 0.1825; 0.8), (0.0400, 0.1000, 0.1800, 0.2300; 1)]$ | 0.2768 |
| $A_{16} = [(1.0000, 1.0000, 1.0000, 1.0000; 1), (1.0000, 1.0000, 1.0000, 1.0000; 1)]$ | 2.0000 |
| $A_{17} = [(1.0000, 1.0000, 1.0000, 1.0000; 1), (1.0000, 1.0000, 1.0000, 1.0000; 1)]$ | 2.0000 |
| $A_{21} = [(0.0000, 0.0000, 0.0000, 0.0000; 1), (0.0000, 0.0000, 0.0000, 0.0000; 1)]$ | 0.0000 |
| $A_{22} = [(0.9475, 0.9850, 0.9925, 0.9925; 0.8), (0.9300, 0.9800, 1.0000, 1.0000; 1)]$ | 1.9647 |
| $A_{23} = [(0.7825, 0.8150, 0.8850, 0.9075; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1)]$ | 1.6968 |
| $A_{24} = [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1)]$ | 0.9835 |
| $A_{25} = [(0.2325, 0.2550, 0.3250, 0.3575; 0.8), (0.1700, 0.2200, 0.3600, 0.4200; 1)]$ | 0.5833 |
| $A_{26} = [(0.2325, 0.2550, 0.3250, 0.3575; 0.8), (0.1700, 0.2200, 0.3600, 0.4200; 1)]$ | 0.5833 |
| $A_{27} = [(0.0875, 0.1200, 0.1600, 0.1825; 0.8), (0.0400, 0.1000, 0.1800, 0.2300; 1)]$ | 0.2768 |
| $A_{31} = [(1.0000, 1.0000, 1.0000, 1.0000; 1), (1.0000, 1.0000, 1.0000, 1.0000; 1)]$ | 2.0000 |
| $A_{32} = [(0.0000, 0.0000, 0.0000, 0.0000; 1), (0.0000, 0.0000, 0.0000, 0.0000; 1)]$ | 0.0000 |
| $A_{33} = [(0.6500, 0.6725, 0.7575, 0.7900; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1)]$ | 1.4333 |
| $A_{34} = [(1.0000, 1.0000, 1.0000, 1.0000; 1), (1.0000, 1.0000, 1.0000, 1.0000; 1)]$ | 2.0000 |
| $A_{35} = [(1.0000, 1.0000, 1.0000, 1.0000; 1), (1.0000, 1.0000, 1.0000, 1.0000; 1)]$ | 2.0000 |
| $A_{36} = [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1)]$ | 0.9835 |
| $A_{37} = [(0.9475, 0.9850, 0.9925, 0.9925; 0.8), (0.9300, 0.9800, 1.0000, 1.0000; 1)]$ | 1.9647 |
| $A_{41} = [(0.7825, 0.8150, 0.8850, 0.9075; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1)]$ | 1.6968 |
| $A_{42} = [(0.0875, 0.1200, 0.1600, 0.1825; 0.8), (0.0400, 0.1000, 0.1800, 0.2300; 1)]$ | 0.2768 |
| $A_{43} = [(0.7825, 0.8150, 0.8850, 0.9075; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1)]$ | 1.6968 |
| $A_{44} = [(0.9475, 0.9850, 0.9925, 0.9925; 0.8), (0.9300, 0.9800, 1.0000, 1.0000; 1)]$ | 1.9647 |
| $A_{45} = [(0.6500, 0.6725, 0.7575, 0.7900; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1)]$ | 1.4333 |
| $A_{46} = [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1)]$ | 0.9835 |
| $A_{47} = [(0.7825, 0.8150, 0.8850, 0.9075; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1)]$ | 1.6968 |

In a similar way, we obtain $\hbar(z_1, z_2) = [(0.3544, 0.3825, 0.4819, 0.5258; 0.8), (0.2908, 0.3407, 0.5417, 0.6451; 1)]$.

By employing the interval type-2 fuzzy PROMETHEE I method, we compute leaving flow $\Phi^+(z_i)$, entering flow $\Phi^-(z_i)$, and their signed distances for each alternative $z_i \in Z$ according to Step 7, as shown in Table 5. Consider $\Phi^+(z_1)$ and $\Phi^-(z_1)$, for example. Applying (29), we obtain

$$\Phi^+(z_1) = \bigoplus_{\beta=2}^{4} \hbar(z_1, z_\beta)$$
$$= \left[ \left( \sum_{\beta=2}^{4} \hbar^L_{11\beta}, \sum_{\beta=2}^{4} \hbar^L_{21\beta}, \sum_{\beta=2}^{4} \hbar^L_{31\beta}, \sum_{\beta=2}^{4} \hbar^L_{41\beta}; \min_{\beta=2}^{4} h^L_{1\beta} \right), \left( \sum_{\beta=2}^{4} \hbar^U_{11\beta}, \sum_{\beta=2}^{4} \hbar^U_{21\beta}, \sum_{\beta=2}^{4} \hbar^U_{31\beta}, \sum_{\beta=2}^{4} \hbar^U_{41\beta}; \min_{\beta=2}^{4} h^U_{1\beta} \right) \right]$$
$$= [(0.3544 + 0.3816 + 0.3797, 0.3825 + 0.4113 + 0.4092, 0.4819 + 0.5218 + 0.5192, 0.5258 + 0.5711 + 0.5682; \min\{0.8, 0.8, 0.8\}), (0.2908 + 0.3113 + 0.3097, 0.3407 + 0.3649 + 0.3630, 0.5417 + 0.5882 + 0.5852, 0.6451 + 0.7030 + 0.6994; \min\{1, 1, 1\})]$$
$$= [(1.1157, 1.2030, 1.5229, 1.6651; 0.8), (0.9118, 1.0686, 1.7151, 2.0475; 1)].$$

**Table 4** The results of preference functions and comprehensive preference indices

| Criteria | Type | $h(A_{1j}, A_{2j})$ | $h(A_{1j}, A_{3j})$ | $h(A_{1j}, A_{4j})$ | $h(A_{2j}, A_{1j})$ | $h(A_{2j}, A_{3j})$ | $h(A_{2j}, A_{4j})$ |
|---|---|---|---|---|---|---|---|
| $x_1$ | V | 0.4420 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $x_2$ | I | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 |
| $x_3$ | IV | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.5000 | 0.0000 |
| $x_4$ | II | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 |
| $x_5$ | VI | 0.2544 | 0.9999 | 0.9847 | 0.0000 | 0.9981 | 0.8954 |
| $x_6$ | VI | 0.0000 | 0.0000 | 0.0000 | 0.9981 | 0.3938 | 0.3938 |
| $x_7$ | III | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 1.0000 | 1.0000 |
| Criteria | Type | $h(A_{3j}, A_{1j})$ | $h(A_{3j}, A_{2j})$ | $h(A_{3j}, A_{4j})$ | $h(A_{4j}, A_{1j})$ | $h(A_{4j}, A_{2j})$ | $h(A_{4j}, A_{3j})$ |
| $x_1$ | V | 1.0000 | 1.0000 | 0.5080 | 1.0000 | 1.0000 | 0.0000 |
| $x_2$ | I | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| $x_3$ | IV | 1.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.5000 |
| $x_4$ | II | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $x_5$ | VI | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6334 |
| $x_6$ | VI | 0.9604 | 0.0000 | 0.0000 | 0.9604 | 0.0000 | 0.0000 |
| $x_7$ | III | 0.0706 | 0.0000 | 0.0000 | 0.6064 | 0.0000 | 0.5358 |

The signed distance-based comprehensive preference index $\hbar(z_\rho, z_\beta)$ of $(z_\rho, z_\beta)$

$\hbar(z_1, z_2) = [(0.3544, 0.3825, 0.4819, 0.5258; 0.8), (0.2908, 0.3407, 0.5417, 0.6451; 1)]$

$\hbar(z_1, z_3) = [(0.3816, 0.4113, 0.5218, 0.5711; 0.8), (0.3113, 0.3649, 0.5882, 0.7030; 1)]$

$\hbar(z_1, z_4) = [(0.3797, 0.4092, 0.5192, 0.5682; 0.8), (0.3097, 0.3630, 0.5852, 0.6994; 1)]$

$\hbar(z_2, z_1) = [(0.2772, 0.3102, 0.4010, 0.4413; 0.8), (0.2163, 0.2721, 0.4556, 0.5555; 1)]$

$\hbar(z_2, z_3) = [(0.5344, 0.5824, 0.7463, 0.8201; 0.8), (0.4284, 0.5136, 0.8448, 1.0189; 1)]$

$\hbar(z_2, z_4) = [(0.4448, 0.4853, 0.6269, 0.6912; 0.8), (0.3534, 0.4258, 0.7119, 0.8627; 1)]$

$\hbar(z_3, z_1) = [(0.3851, 0.4246, 0.5302, 0.5748; 0.8), (0.3147, 0.3802, 0.5937, 0.7068; 1)]$

$\hbar(z_3, z_2) = [(0.1531, 0.1662, 0.2032, 0.2187; 0.8), (0.1288, 0.1506, 0.2255, 0.2643; 1)]$

$\hbar(z_3, z_4) = [(0.0778, 0.0844, 0.1032, 0.1111; 0.8), (0.0654, 0.0765, 0.1145, 0.1343; 1)]$

$\hbar(z_4, z_1) = [(0.4095, 0.4524, 0.5702, 0.6209; 0.8), (0.3310, 0.4029, 0.6410, 0.7681; 1)]$

$\hbar(z_4, z_2) = [(0.1531, 0.1662, 0.2032, 0.2187; 0.8), (0.1288, 0.1506, 0.2255, 0.2643; 1)]$

$\hbar(z_4, z_3) = [(0.3087, 0.3349, 0.4257, 0.4664; 0.8), (0.2502, 0.2967, 0.4803, 0.5762; 1)]$

Then, the signed distance $d(\Phi^+(z_1), \tilde{0}_1) = 2.8112$. Next, applying (30), we obtain

$$\begin{aligned}\Phi^-(z_1) &= \bigoplus_{\rho=2}^{4} \hbar(z_\rho, z_1)\\ &= \left[\left(\sum_{\rho=2}^{4}\hbar^L_{1\rho 1}, \sum_{\rho=2}^{4}\hbar^L_{2\rho 1}, \sum_{\rho=2}^{4}\hbar^L_{3\rho 1}, \sum_{\rho=2}^{4}\hbar^L_{4\rho 1}; \min_{\rho=2}^{4} h^L_{\rho 1}\right),\right.\\ &\quad \left.\left(\sum_{\rho=2}^{4}\hbar^U_{1\rho 1}, \sum_{\rho=2}^{4}\hbar^U_{2\rho 1}, \sum_{\rho=2}^{4}\hbar^U_{3\rho 1}, \sum_{\rho=2}^{4}\hbar^U_{4\rho 1}; \min_{\rho=2}^{4} h^U_{\rho 1}\right)\right]\\ &= [(0.2772 + 0.3851 + 0.4095, 0.3102 + 0.4246\\ &\quad + 0.4524, 0.4010 + 0.5302 + 0.5702, 0.4413 + 0.5748\\ &\quad + 0.6209; \min\{0.8, 0.8, 0.8\}), (0.2163 + 0.3147\\ &\quad + 0.3310, 0.2721 + 0.3802 + 0.4029, 0.4556 + 0.5937\\ &\quad + 0.6410, 0.5555 + 0.7068 + 0.7681; \min\{1, 1, 1\})]\\ &= [(1.0718, 1.1872, 1.5014, 1.6370; 0.8), (0.8620,\\ &\quad 1.0552, 1.6903, 2.0304; 1)].\end{aligned}$$

The corresponding signed distance $d(\Phi^-(z_1), \tilde{0}_1)$=2.7632.

In Step 8 of PROMETHEE I, we obtain the results of $z_1 \succ^+ z_3$, $z_1 \succ^+ z_4$, $z_2 \succ^+ z_1$, $z_2 \succ^+ z_3$, $z_2 \succ^+ z_4$, and $z_4 \succ^+ z_3$ using $d(\Phi^+(z_i), \tilde{0}_1) > d(\Phi^+(z_j), \tilde{0}_1)$ for $z_i, z_j \in Z$. Similarly, we obtain the results of $z_1 \succ^- z_3$, $z_2 \succ^- z_1$, $z_2 \succ^- z_3$, $z_2 \succ^- z_4$, $z_4 \succ^- z_1$, and $z_4 \succ^- z_3$ using $d(\Phi^-(z_i), \tilde{0}_1) < d(\Phi^-(z_j), \tilde{0}_1)$ for $z_i, z_j \in Z$. Following the procedure using $(\succ^{\mathrm{I}}, \sim^{\mathrm{I}}, \mathbb{R})$, we produce the interval type-2 fuzzy PROMETHEE I partial preorders $z_1 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_1$, $z_2 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_4$, $z_4 \succ^{\mathrm{I}} z_3$, and $z_1 \mathbb{R} z_4$. These partial preorders are represented in Fig. 4. The best choice is $z_2$.

By applying the interval type-2 fuzzy PROMETHEE II method, we calculate the net flow $\Phi(z_i)$ and its signed distance $d(\Phi(z_i), \tilde{0}_1)$ for each alternative $z_i \in Z$ according to

**Table 5** The results of leaving flows, entering flows, and net flows

| | |
|---|---|
| The leaving flow $\Phi^+(z_i)$ of alternative $z_i \in Z$ | $d(\Phi^+(z_i), \tilde{0}_1)$ |
| $\Phi^+(z_1) = [(1.1157, 1.2030, 1.5229, 1.6651; 0.8), (0.9118, 1.0686, 1.7151, 2.0475; 1)]$ | 2.8112 |
| $\Phi^+(z_2) = [(1.2564, 1.3779, 1.7742, 1.9526; 0.8), (0.9981, 1.2115, 2.0123, 2.4371; 1)]$ | 3.2553 |
| $\Phi^+(z_3) = [(0.6160, 0.6752, 0.8366, 0.9046; 0.8), (0.5089, 0.6073, 0.9337, 1.1054; 1)]$ | 1.5495 |
| $\Phi^+(z_4) = [(0.8713, 0.9535, 1.1991, 1.3060; 0.8), (0.7100, 0.8502, 1.3468, 1.6086; 1)]$ | 2.2133 |
| The entering flow $\Phi^-(z_i)$ of alternative $z_i \in Z$ | $d(\Phi^-(z_i), \tilde{0}_1)$ |
| $\Phi^-(z_1) = [(1.0718, 1.1872, 1.5014, 1.6370; 0.8), (0.8620, 1.0552, 1.6903, 2.0304; 1)]$ | 2.7632 |
| $\Phi^-(z_2) = [(0.6606, 0.7149, 0.8883, 0.9632; 0.8), (0.5484, 0.6419, 0.9927, 1.1737; 1)]$ | 1.6468 |
| $\Phi^-(z_3) = [(1.2247, 1.3286, 1.6938, 1.8576; 0.8), (0.9899, 1.1752, 1.9133, 2.2981; 1)]$ | 3.1194 |
| $\Phi^-(z_4) = [(0.9023, 0.9789, 1.2493, 1.3705; 0.8), (0.7285, 0.8653, 1.4116, 1.6964; 1)]$ | 2.2999 |
| The net flow $\Phi(z_i)$ of alternative $z_i \in Z$ | $d(\Phi(z_i), \tilde{0}_1)$ |
| $\Phi(z_1) = [(-0.5213, -0.2984, 0.3357, 0.5933; 0.8), (-1.1186, -0.6217, 0.6599, 1.1855; 1)]$ | 0.0481 |
| $\Phi(z_2) = [(0.2932, 0.4896, 1.0593, 1.2920; 0.8), (-0.1756, 0.2188, 1.3704, 1.8887; 1)]$ | 1.6084 |
| $\Phi(z_3) = [(-1.2416, -1.0186, -0.4920, -0.3201; 0.8), (-1.7892, -1.3060, -0.2415, 0.1155; 1)]$ | −1.5699 |
| $\Phi(z_4) = [(-0.4992, -0.2958, 0.2202, 0.4037; 0.8), (-0.9864, -0.5614, 0.4815, 0.8801; 1)]$ | −0.0866 |

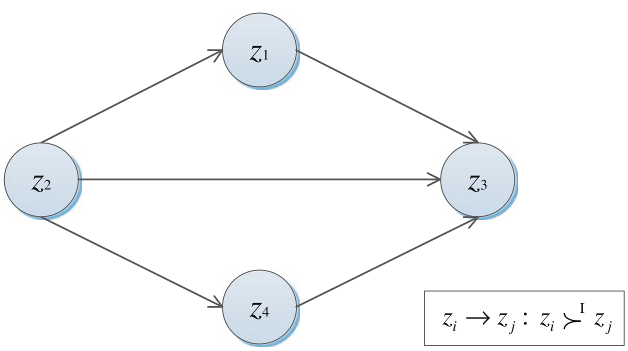


**Fig. 4** The partial preorder in the landfill siting problem

Step 7, as shown in Table 5. Consider $\Phi(z_1)$ as an example. Applying (31), we obtain

$$\begin{aligned}\Phi(z_1) &= (\Phi^+(z_1))\Theta(\Phi^-(z_1))\\ &= [(1.1157 - 1.6370, 1.2030 - 1.5014, 1.5229 - 1.1872,\\ &\quad 1.6651 - 1.0718; \min\{0.8, 0.8\}), (0.9118 - 2.0304,\\ &\quad 1.0686 - 1.6903, 1.7151 - 1.0552, 2.0475 - 0.8620;\\ &\quad \min\{1, 1\})]\\ &= [(-0.5213, -0.2984, 0.3357, 0.5933; 0.8),\\ &\quad (-1.1186, -0.6217, 0.6599, 1.1855; 1)].\end{aligned}$$

The corresponding signed distance is $d(\Phi(z_1), \tilde{0}_1) = 0.0481$.

In Step 8 of PROMETHEE II, we obtain $d(\Phi(z_2), \tilde{0}_1) > d(\Phi(z_1), \tilde{0}_1) > d(\Phi(z_4), \tilde{0}_1) > d(\Phi(z_3), \tilde{0}_1)$. Therefore, we obtain the interval type-2 fuzzy PROMETHEE II complete preorders as follows: $z_2 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_3$. The complete preorder is represented in Fig. 5, and the best choice is $z_2$.

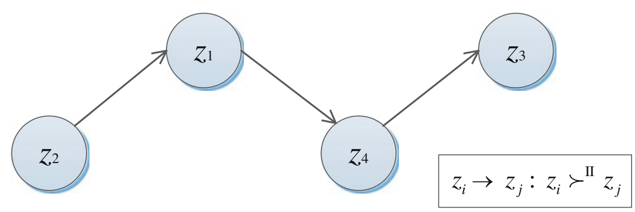


**Fig. 5** The complete preorder in the landfill siting problem

## 4.2 Discussion

The result of the complete preorder produced by the interval type-2 fuzzy PROMETHEE II method can facilitate decision making in general. However, the partial preorder contains more realistic information, which can often be useful in decision making, especially with regard to incomparability (Brans et al. 1984, 1986).

The leaving flow is a positive outranking flow (Vinodh and Jeya Girubha 2012). It measures the degree of dominance to which a specific alternative outranks all other alternatives. Conversely, the entering flow, which is also called a negative outranking flow, measures the degree to which a specific alternative is dominated by all other alternatives. Therefore, the leaving and entering flows can be regarded as positive and negative information, respectively, about the outranking relations.

There may be asymmetry in the effects of positive and negative information on outranking relations. Cacioppo et al.

(1997) and Grabisch et al. (2008) indicated that negative information has more weight than positive information. That is, negative information has a much stronger impact on outranking relations than positive information, and this phenomenon is consistent with the operation of a negativity bias (Cacioppo et al. 1997). Considering the illustrative problem of landfill site selection, the conflict occurs in the outranking relations between alternatives $z_1$ and $z_4$: $z_1 \succ^{+} z_4$ based on leaving flows, and $z_4 \succ^{-} z_1$ based on entering flows. If the negativity bias exists, the decision maker will attach more importance to the entering flow than to the leaving flow. Thus, the decision maker tends to accept the result of $z_4 \succ z_1$ rather than $z_1 \mathbb{R} z_4$. Nevertheless, the complete preorder obtained by the interval type-2 fuzzy PROMETHEE II method shows $z_1 \succ^{\mathrm{II}} z_4$. Because we can observe the separate ranking results yielded by the entering and leaving flows, the interval type-2 fuzzy PROMETHEE I method is able to provide much more detailed outcomes for facilitating a realistic decision-making process.

The personal characteristics of the decision maker may also influence the subjective judgments of positive and negative information. For example, Chernev (2004) distinguished between two types of people, i.e., people with promotion focus and people with prevention focus, on the basis of their personal goal orientation. People with promotion focus are primarily interested in their own growth and development, have more hopes and aspirations, and favor the presence of positive outcomes. In contrast, persons with prevention focus are primarily interested in safety and security, are more concerned with duties and obligations, and favor the absence of negative outcomes (Chernev 2004; Pham et al. 2004). Therefore, the influence of a negativity bias should be more prominent in prevention-focused individuals than in promotion-focused individuals. Moreover, positive outcomes will have more weight than negative outcomes for a decision maker with a promotion focus. Based on this discussion, a prospective development is suggested to incorporate the relative weight or worth of the positive part (i.e., leaving flow) and negative part (i.e., entering flow) into the calculation of the net flow. That is, we can define a parameterized net flow that represents a mixed result of the entering and leaving flows.

### 4.3 Comparative analysis

A comparative study was conducted to validate the results of the proposed method with those from ordinary fuzzy PROMETHEE methods. To compare the solution results on a common basis, we used the same linguistic rating data in Table 2 to solve the problem of landfill site selection.

As mentioned before, this study adopted a nine-point rating scale, which originates from Chen's (1996) work, to measure the variability in responses for better sensitivity. Chen's (1996) used a nine-member linguistic term set (including absolutely low, very low, low, medium low, medium, medium high, high, very high, and absolutely high) based on Chen (1988) to represent the linguistic terms. Additionally, he presented the corresponding trapezoidal fuzzy numbers for each linguistic term. In this paper, we employ Chen's (1996) nine translations to convert linguistic terms into upper trapezoidal fuzzy numbers of IT2TrFNs. A detailed exposition about the nine translations of linguistic terms into IT2TrFNs has been presented in Chen (2011a,b, 2012a). In the following comparative analysis, we employ Chen's (1996) nine translations to convert linguistic terms in Table 2 into trapezoidal fuzzy numbers, and then we solve the MCDA problem of landfill sites using the ordinary fuzzy PROMETHEE method.

In addition to Chen's (1996) translation standards, we convert the IT2TrFN ratings in Table 3 into mean trapezoidal fuzzy numbers. Recall that the IT2TrFN rating of alternative $z_i$ with respect to criterion $x_j$ is expressed as $[(a^L_{1ij}, a^L_{2ij}, a^L_{3ij}, a^L_{4ij}; h^L_{ij}), (a^U_{1ij}, a^U_{2ij}, a^U_{3ij}, a^U_{4ij}; h^U_{ij})]$. The corresponding mean trapezoidal fuzzy number is defined as follows: $((a^L_{1ij} + a^U_{1ij})/2, (a^L_{2ij} + a^U_{2ij})/2, (a^L_{3ij} + a^U_{3ij})/2, (a^L_{4ij} + a^U_{4ij})/2)$. Consider the problem of landfill site selection within an ordinary fuzzy environment. The linguistic rating data in Table 2 can be expressed as ordinary fuzzy numbers by using Chen's (1996) trapezoidal fuzzy numbers or the mean trapezoidal fuzzy numbers.

We implement the ordinary fuzzy PROMETHEE method, which relies on Euclidean distances to solve the problem of landfill site selection. Furthermore, we employ the proposed interval type-2 fuzzy PROMETHEE methods to handle the ordinary fuzzy data. Table 6 reveals the obtained results consisting of the partial and complete ranking orders of the candidate locations using PROMETHEE I and II, respectively. The proposed methods yield the same ranking results when coping with the rating data using Chen's (1996) trapezoidal fuzzy numbers, the mean trapezoidal fuzzy numbers, and the IT2TrFNs. Thus, we can conclude that the proposed interval type-2 fuzzy PROMETHEE I and II methods can be effectively applied to the ordinary fuzzy environment as well. When the ordinary fuzzy PROMETHEE I method is used, the partial ranking results of the candidate locations are the same as those obtained by using the proposed methods. Regarding the results yielded by the fuzzy PROMETHEE II method, the complete order of the candidate locations is $z_2 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_3$, which is similar to the result ($z_2 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_3$) that is obtained using the proposed methods. As revealed in the comparative result, the proposed methods can be easily adapted to the ordinary fuzzy or interval type-2 fuzzy environments. The potential of the proposed methods for practical applications is validated using this comparative analysis.

**Table 6** Comparison analysis of the obtained results

| Method | Partial ranking | Complete ranking |
|---|---|---|
| Fuzzy PROMETHEE | | |
| Chen's (1996) trapezoidal fuzzy numbers | $z_1 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_1$, $z_2 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_4$, $z_4 \succ^{\mathrm{I}} z_3$, $z_1 \mathbb{R} z_4$ | $z_2 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_3$ |
| Mean trapezoidal fuzzy numbers | $z_1 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_1$, $z_2 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_4$, $z_4 \succ^{\mathrm{I}} z_3$, $z_1 \mathbb{R} z_4$ | $z_2 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_3$ |
| The proposed method | | |
| Chen's (1996) trapezoidal fuzzy numbers | $z_1 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_1$, $z_2 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_4$, $z_4 \succ^{\mathrm{I}} z_3$, $z_1 \mathbb{R} z_4$ | $z_2 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_3$ |
| Mean trapezoidal fuzzy numbers | $z_1 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_1$, $z_2 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_4$, $z_4 \succ^{\mathrm{I}} z_3$, $z_1 \mathbb{R} z_4$ | $z_2 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_3$ |
| IT2TrFNs | $z_1 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_1$, $z_2 \succ^{\mathrm{I}} z_3$, $z_2 \succ^{\mathrm{I}} z_4$, $z_4 \succ^{\mathrm{I}} z_3$, $z_1 \mathbb{R} z_4$ | $z_2 \succ^{\mathrm{II}} z_1 \succ^{\mathrm{II}} z_4 \succ^{\mathrm{II}} z_3$ |

## 5 Conclusions

In this paper, we developed interval type-2 fuzzy PROMETHEE I and II methods to manage the MCDA problems in the context of IT2TrFNs. This study has extended the definitions of classical generalized criteria to propose the signed distance-based generalized criteria based on IT2FSs. According to the signed distance-based usual criterion, U-shaped criterion, V-shaped criterion, level criterion, V-shaped with indifference criterion, and Gaussian criterion, we can determine the preference function and further derive the signed distance-based comprehensive preference index by combining the criterion importance. To evaluate the alternatives using the outranking relation, we employed the concepts of leaving flows, entering flows, and net flows to develop the procedures for partial preordering and complete preordering of the alternatives. The feasibility and applicability of the proposed interval type-2 fuzzy PROMETHEE I and II methods have been validated using the practical MCDA problem of landfill site selection.

Preliminary research on the development of PROMETHEE methodologies using IT2FSs has been conducted in this study. The effectiveness of the proposed methods was supported by the illustrative calculations. Note that this paper does not intend to replace ordinary fuzzy PROMETHEE with interval type-2 fuzzy PROMETHEE methods. Because human judgment is often vague under many conditions, the available information is sometimes insufficient for determining an exact definition of the degree of membership for certain elements. IT2FSs with interval-type membership grades are appropriate for dealing with such situations. Thus, the proposed methods can be deemed a complement to the existing PROMETHEE methodologies. Future studies can focus on the potential for extending other methodologies of the PROMETHEE family to the interval type-2 fuzzy environment.

## Appendix

**Definition 6.1** Let $X$ be an ordinary finite nonempty set. Let Int([0, 1]) denote a set of all closed subintervals of [0, 1]. The mapping $A$: $X \to$ Int([0, 1]) is known as an IT2FS on $X$. All IT2FSs on $X$ are denoted by IT2FS$(X)$.

**Definition 6.2** If $A \in$ IT2FS$(X)$, let $A(x) = [A^L(x), A^U(x)]$, where $x \in X$ and $0 \leq A^L(x) \leq A^U(x) \leq 1$. The two T1FSs $A^L : X \to [0, 1]$ and $A^U : X \to [0, 1]$ are known as the lower and upper fuzzy sets, respectively, with respect to $A$. If $A(x)$ is convex and defined on a closed and bounded interval, then $A$ is known as "an interval type-2 fuzzy number on $X$".

**Definition 6.3** Let $A^L(= (a_1^L, a_2^L, a_3^L, a_4^L; h_A^L))$ and $A^U(= (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U))$ be the lower and upper trapezoidal fuzzy numbers defined on the universe of discourse $X$, where $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L$, $a_1^U \leq a_2^U \leq a_3^U \leq a_4^U$, $0 \leq h_A^L \leq h_A^U \leq 1$, $a_1^U \leq a_1^L$, $a_4^L \leq a_4^U$, and $A^L \subset A^U$. Let $\xi \in \{L, U\}$. The membership function of $A^\xi$ for each $\xi$ is expressed as follows:

$$A^\xi(x) = \begin{cases} h_A^\xi(x - a_1^\xi)/(a_2^\xi - a_1^\xi) & \text{for } a_1^\xi \leq x \leq a_2^\xi, \\ h_A^\xi & \text{for } a_2^\xi \leq x \leq a_3^\xi, \\ h_A^\xi(a_4^\xi - x)/(a_4^\xi - a_3^\xi) & \text{for } a_3^\xi \leq x \leq a_4^\xi, \\ 0 & \text{otherwise.} \end{cases} \tag{36}$$

An IT2TrFN $A$ on $X$ is represented by the following:

$$A = [A^L, A^U] = \left[(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)\right]. \quad (37)$$

The extension principle (Zadeh 1975) can be employed to develop fuzzy arithmetic defined as IT2FSs (Aisbett et al. 2010; Gilan et al. 2012). Let $\oplus$ denote the addition operation, and let $A$ and $B$ denote IT2FSs. By using Zadeh's extension principle, we define an IT2FS for a set of all real numbers $A \oplus B$ with the following equation:

$$(A \oplus B)(z) = \sup_{z=x+y} \min [A(x), B(y)], \quad (38)$$

where sup is the supremum. Based on interval-valued arithmetic, standard arithmetic operations on trapezoidal-shaped fuzzy numbers can be extended to IT2TrFNs.

**Definition 6.4** Let $A$ and $B$ be two nonnegative IT2TrFNs. $A = [(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)]$, and $B = [(b_1^L, b_2^L, b_3^L, b_4^L; h_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; h_B^U)]$ on $X$. The arithmetic operations on $A$ and $B$ are defined as follows:

$$A \oplus B = \Big[\Big(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\{h_A^L, h_B^L\}\Big), \Big(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min\{h_A^U, h_B^U\}\Big)\Big]; \quad (39)$$

$$A \ominus B = \Big[\Big(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min\{h_A^L, h_B^L\}\Big), \Big(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; \min\{h_A^U, h_B^U\}\Big)\Big]; \quad (40)$$

$$A \otimes B = \Big[\Big(a_1^L \cdot b_1^L, a_2^L \cdot b_2^L, a_3^L \cdot b_3^L, a_4^L \cdot b_4^L; \min\{h_A^L, h_B^L\}\Big), \Big(a_1^U \cdot b_1^U, a_2^U \cdot b_2^U, a_3^U \cdot b_3^U, a_4^U \cdot b_4^U; \min\{h_A^U, h_B^U\}\Big)\Big]; \quad (41)$$

$$A \oslash B = \Big[\Big(a_1^L/b_4^L, a_2^L/b_3^L, a_3^L/b_2^L, a_4^L/b_1^L; \min\left(h_A^L, h_B^L\right)\Big), \Big(a_1^U/b_4^U, a_2^U/b_3^U, a_3^U/b_2^U, a_4^U/b_1^U; \min\left(h_A^U, h_B^U\right)\Big)\Big], b_1^L, b_2^L, b_3^L, b_4^L, b_1^U, b_2^U, b_3^U, b_4^U \neq 0; \quad (42)$$

$$q \cdot A = A \cdot q = \begin{cases} \Big[\Big(q \cdot a_1^L, q \cdot a_2^L, q \cdot a_3^L, q \cdot a_4^L; h_A^L\Big), \Big(q \cdot a_1^U, q \cdot a_2^U, q \cdot a_3^U, q \cdot a_4^U; h_A^U\Big)\Big] & \text{if } q \geq 0, \\ \Big[\Big(q \cdot a_4^L, q \cdot a_3^L, q \cdot a_2^L, q \cdot a_1^L; h_A^L\Big), \Big(q \cdot a_4^U, q \cdot a_3^U, q \cdot a_2^U, q \cdot a_1^U; h_A^U\Big)\Big] & \text{if } q \leq 0; \end{cases} \quad (43)$$

$$A/q = \begin{cases} \Big[\Big(\frac{a_1^L}{q}, \frac{a_2^L}{q}, \frac{a_3^L}{q}, \frac{a_4^L}{q}; h_A^L\Big), \Big(\frac{a_1^U}{q}, \frac{a_2^U}{q}, \frac{a_3^U}{q}, \frac{a_4^U}{q}; h_A^U\Big)\Big] & \text{if } q > 0, \\ \Big[\Big(\frac{a_4^L}{q}, \frac{a_3^L}{q}, \frac{a_2^L}{q}, \frac{a_1^L}{q}; h_A^L\Big), \Big(\frac{a_4^U}{q}, \frac{a_3^U}{q}, \frac{a_2^U}{q}, \frac{a_1^U}{q}; h_A^U\Big)\Big] & \text{if } q < 0. \end{cases} \quad (44)$$

The multiplication and division operations produce approximate IT2TrFNs for simple computations.

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